

*Resolve to remain loyal to the Association by paying your dues on time, reading the Journal, writing articles about your experiences and trying to interest fellow teachers in the Association.*—  
WALTER H. CARNAHAN.

## CONTENTS FOR APRIL, 1945

If May Were November— <i>Walter H. Carnahan</i> .....	297
The Science Teaching Film Comes to the Elementary Grades— <i>W. A. Witlich</i> .....	298
Scientific Interdigitation— <i>Frederick F. Yonkman</i> .....	301
An Experiment on the Banking of Curves— <i>Walter V. Burg</i> .....	307
Miniature Double Stars— <i>E. M. Tingley</i> .....	310
Illustrating the Conic Sections— <i>Walter H. Carnahan</i> .....	313
Pandagonal Magic Squares on Square Bases and Their Transformations— <i>Hiram B. Loomis</i> .....	315
Book Clubs for Professional Reading— <i>J. R. Mayor</i> .....	323
Diagrams for Specific Heat and Change of State Problems— <i>H. F. Cope</i> .....	326
Indian Medicine— <i>John S. Wright</i> .....	329
Teaching Techniques for Slower Students in Army College Training Programs— <i>G. D. McGrath</i> .....	335
Theory of Iterated Trigonometric Functions. Part I. The Iterated Sine— <i>W. Raymond Griffin</i> .....	341
Our Warty Assistants— <i>Percy A. Morris</i> .....	351
A Historian Views Science— <i>Louis Martin Sears</i> .....	356
Relations of Trigonometric Functions— <i>Leonard Thatcher</i> .....	365
Science in a New World— <i>Andrew Leitch</i> .....	367
Notes from a Mathematics Classroom— <i>Joseph A. Nyberg</i> .....	372
Eastern Association of Physics Teachers— <i>Carl W. Staples</i> .....	376
The Physical-Chemical Characteristics of Some of the Proteins of Human Plasma (Abstract)— <i>L. E. Strong</i> .....	377
Dehydration of Foods (Abstract)— <i>S. C. Prescott</i> .....	378
Problem Department— <i>G. H. Jamison</i> .....	381
Book Reviews .....	386

## Superior Texts Bring Results!

\* \* \*

### HEALTH FOR YOU

By Crisp. This modern health text is refreshingly new in its content and its presentation of the subject of health.

\$1.80 list price

### PHYSICS WORKBOOK

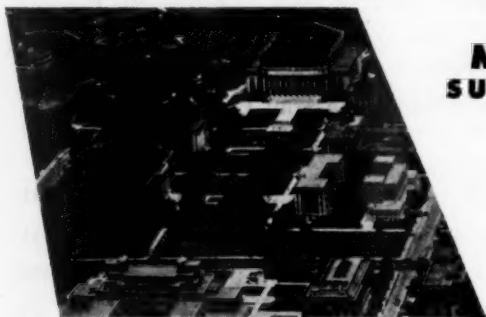
By Buell and Schuler. This up-to-date combination workbook and laboratory manual is designed for the average, everyday high school physics course. Included are page references to the ten latest physics textbooks.

\$1.20 list price

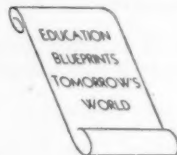
*Send for descriptive circulars or examination copies*

**J. B. Lippincott Company**

Philadelphia  
Chicago • New York



### MINNESOTA SUMMER SESSION



**I**N ADDITION to more than 750 subjects of study covering all fields of educational and scientific interest, especial emphasis is being placed on wartime as well as post-war educational needs. A teaching staff of 600 persons including educators of national and international reputation—plus the splendid

facilities of great libraries and laboratories—offer outstanding opportunity for both graduate and undergraduate study. There will be special courses and workshops for teachers in primary, secondary and higher education. Counseling facilities for returning veterans.

*First term registration, Monday and Tuesday, June 18 and 19. Second term registration, Monday, July 30. Write now for complete bulletin. Director of Summer Session, 752 Administration Building.*

**UNIVERSITY OF MINNESOTA**

MINNEAPOLIS 14, MINNESOTA

*75th Year of Service*

**Please Mention School Science and Mathematics when answering Advertisements**

# SCHOOL SCIENCE AND MATHEMATICS

---

VOL. XLV

APRIL, 1945

WHOLE NO. 393

---

## IF MAY WERE NOVEMBER

WALTER H. CARNAHAN

*President of Central Association of Science and Mathematics Teachers*

A number of members of Central Association have asked me the question, "Will there be a meeting in 1945?" No doubt many who have not asked the question are concerned about this matter. This includes your president. May I take you into my confidence and present the picture as I see it at this time?

If our meeting were scheduled to be held in May rather than in November, I think there would be no doubt that we should be compelled to pass this year without a meeting. And we would all yield to the necessity willingly if not gladly. There is not a member of Central Association who would want to occupy a seat on a train if some service man or woman would be deprived of the chance to go home on furlough or to return to duty after a visit home. None of us would want to occupy a hotel room needed by a service man who would be compelled to sleep in a chair at a U. S. O. center.

However, our annual meeting is not scheduled to be held until November, and it is quite possible that by November 23 conditions may be quite different from those obtaining today. Let us hope this may be true. We are taking steps to adjust to conditions as best we can if it is not possible to meet as usual and hoping such adjustment will not be necessary. In the meantime, plans are going forward for the meeting in full confidence that it will be held. Committees are being appointed, section chairmen are going about their duties, speakers are being secured, and arrangements for a meeting place are being pushed

along. Please make your plans to attend the annual convention and trust that they can be carried out. And above all, resolve to remain loyal to the Association by paying your dues on time, reading the Journal, writing articles about your experiences and trying to interest fellow teachers in the Association. I have full confidence that "We'll be seeing you."

---

### THE SCIENCE TEACHING FILM COMES TO THE ELEMENTARY GRADES\*

W. A. WITTICH

*Acting Director, Bureau of Visual Instruction, Extension Division,  
University of Wisconsin, Madison, Wis.*

The trial-and-error development and use of motion pictures for schools has been so confused that the first responsibility of the teacher or supervisor contemplating the use in the classroom of authentic *sound text films* is to establish definitely a working definition which will guide in their selection and utilization. First of all, the text film is one which presents learning situations which can not be duplicated in the classroom under any other circumstances than true motion-picture photography accompanied by appropriate sound effects. To produce films which merely duplicate good teaching procedures already developed and in use in the classroom would obviously then be a waste of time and money. Only those things which cannot be produced under any traditional circumstances should be developed through the medium of the sound film.

Supposing, then, that we are very careful in our selection of the teaching film. What advantages may we claim for it? Many subjective evaluations have been made concerning its usefulness but many of them, particularly those of adverse nature, have been made in evaluation of the motion-picture film which should never have been made, which was produced merely because it covered some spectacular though non-utilitarian area and was used during those days of first enthusiasm for the film in the classroom. The true teaching film, however, has estab-

---

\* The Bureau of Visual Instruction at the University of Wisconsin has recently instituted a service through which any Wisconsin or Midwestern State science teacher has but to submit a list of the studies contemplated; the curriculum department will recommend the authentic teaching films that will make a distinct learning contribution to the area.

lished itself through sound research to make contributions such as these to the classroom:

1. The true text or teaching film assists in greater retention of factual information.

2. Over long periods of time rather than short, information learned via the teaching film is retained increasingly.

3. When children see first hand the actual process, they are studying and experience through observation and sound reception, their learning is accomplished with great accuracy, with great vividness, and with authenticity.

4. Research studies also point very definitely to motivation being given reading comprehension and general oral facility in those situations where the teaching or text film is used.

The question is often asked: Of what specific use is the science teaching film at the various levels in the elementary school? At the primary level, the greatest single contribution which can be claimed is that of establishing visual imagery which serves as a background for both reading and language arts. The readiness attained by viewing such films as *The Black Bear Twins*, *Shep the Farm Dog*, *Robin Redbreast*, *Elephants*, *Animals of Our Woods*, and other splendidly conceived pictures for children of primary-grade interest and ability is without question. Too often we attempt to lead children into reading situations and into oral expression situations before we have built within them backgrounds of experience which will be conducive to understanding in the reading situation, or, expression in the language arts.

At the intermediate level, where natural science phenomena become increasingly complex, it is all the more necessary that we bring to children authentic experiences which they can observe on an actual "being there" basis. To allow children to see a natural phenomenon reconstructed in the classroom through the sound film enables them not only to witness an experience but further allows them to recapture or re-see this experience as many times as is necessary in order that they actually understand completely and meaningfully the true nature, causes and effects of life as it exists about them. The film reconstructs experiences in nature. Better than that, it allows the teacher to recall this visual experience merely by reshowing the film.

That it is necessary to anticipate, to show the film, to discuss the film, and to reshow it is very easily established by conducting any group of elementary school children through a motion-

picture film learning experience after which they are given an opportunity to evaluate what they saw in terms of some self-evaluation or self-inventory technique.\*

Too often when the teacher is using the sound film, she is so struck by the spectacular contribution it makes that she is prone to relax her usual teaching vigil and to place too much confidence in the film itself to produce desired learning outcomes. This must never be the case. Established principles of learning similar to those used in traditional teaching situations are just as necessary in the correct method of utilization of the sound film as they are in other areas of school work. The teacher who uses the sound film in the classroom must remember that here, just as in other teaching areas, children learn best when they are challenged, children learn best when their curiosities are aroused, children learn best when hazards to their learning are removed insofar as possible, and children learn best when responsibilities which they are able to accomplish are clearly set up.

In summary, the true text film is perhaps the most outstanding, most vivid, most authentic supplement to classroom instruction that has been developed since the turn of the century. It must never be supposed that it will do the whole teaching job. To use it correctly is hard work both on the part of the teacher and of the pupil. In return, however, it will pay learning dividends beyond all contemplation. The thinking teacher will investigate the text film, will take great care in selecting only those which are true text films. She will avoid using great numbers of them, will restrict her use to those few which make a definite contribution to learning experience in the classroom, and will use those few with full realization that tried and tested tenets of psychology and classroom method are just as necessary to effective utilization of the teaching film as they are in any other area of instruction.

---

\* What is spoken of here is the type of guided learning experience which is available through such prepared teaching film study guides as the Visual Learning Guides produced and distributed by the National Audio-Visual Council, 160 North La Salle Street, Chicago.

---

The future of the world is left to highly educated races who alone can handle the scientific apparatus necessary for preeminence in peace or survival in war. I hope our education will become broader and more liberal. All wisdom is not new wisdom and the past should be studied if the future is to successfully encountered. . . .

WINSTON CHURCHILL

## SCIENTIFIC INTERDIGITATION\*

FREDERICK F. YONKMAN

*Ciba Pharmaceutical Products Co., Inc., Lafayette Park, Summit,  
New Jersey*

Someone has said that teachers are born, not made. Whether true or not, all of us must agree that a teacher can improve, and usually does so with experience, the more so if he desires intensely to improve. Regardless of training as a teacher, one usually becomes a good teacher if one is profoundly interested in his subject and is conscientious and sincere in his effort to inspire others. For that is what teaching really is—not the *instillation of learning* but the *inspiration to learn*. If we can but inspire others to be more intense in their search of knowledge, we will be successful teachers. Perhaps the greatest preceptor, with whom it was my good fortune to be associated as his student, was Doctor Frank Patterson of Hope College in Holland, Michigan. This good man, who has long since gone to his reward, could not give a formal lecture, either to his own or his students' satisfaction, yet, in the classroom as recitation periods developed into discussion groups, the urge to learn was consciously or unconsciously imparted to us from contacts with this superb teacher. Botany was of interest, perhaps for botany's sake, but the more so because of "Patty's" recital of Tennyson's "Flower in the Crannied Wall—." Or again, because of his dramatization, subtly performed and almost too modestly, of the interplay of those chemical and physical phenomena involved in the production of carbohydrate by the green plant pigment, chlorophyl. Under the inspiring influence of Doctor Patterson the latent was aroused and the alert accelerated; study was fascinating, not only essential—it was challenging. And that is your mission and mine as teachers of science to any group regardless of age or class of undergraduate or graduate students, as long as there is in them the will to learn. It is our privilege to inspire, their responsibility and opportunity to learn.

As a teacher of medicine I have had some most interesting experiences. One of these has been to assist in the selection of potential freshman students from numerous undergraduate schools scattered far and wide. One of the stock questions asked every candidate was "Why do you want to study medicine and

\* An address before the Senior High School group of the Central Association of Science and Mathematics Teachers at Chicago, November 25, 1944.

when did you so decide?" The responses were varied but most consistent was the statement that "I liked this subject or that." When further pressed, it was frequently volunteered that the student enjoyed a certain subject because he appreciated the manner in which that subject was introduced and presented. That is the secret! So you see, as a member of an admissions committee of a medical college, my share in choosing future physicians was indeed a minor role—you were doing the selection for us, you were picking your own physician even then but perhaps unwittingly. You inspired the student to study medicine, the professor of medicine merely assisted in the development of his fundamental eagerness regarding medicine; before long your selection had matured. And so it is in every profession—the student is inspired to enter certain professional fields of activity. As a teacher of science you have the great responsibility of correlating one science with another so that those entrusted to your tutoring may see the forest as well as the trees. Yours is a rare opportunity and privilege. By scientific interdigitation or relating one field of science to another, the teacher progresses far along the road of inspiring one to learn.

For centuries custodians of the public health have alleviated man's pains with morphine. This valuable drug is obtained from a crude drug source known as opium which in turn is offered by nature in the gourd of the oriental poppy. Opium itself can be administered in crude drug form, either as a powder or as an alcoholic tincture, but the pharmacist has given us, with the help of the chemist, the most important alkaloids found in opium by various methods of extraction of its active principles. Among these alkaloids one finds morphine, codeine, papaverine, and several others of less importance in the fields of medicine. Requisite to this development have been the science of botany in recognition by the pharmacognocist of which plants to select as a source of opium, and the science of chemistry in the pharmaceutical extraction and identification of the important alkaloids. And finally the pharmacologist steps into the picture; it is his responsibility to study the various reactions of living tissues to these alkaloids. He is in reality a biotrepist. In order to carry out his duties properly he must be familiar with chemistry, physiology, anatomy, pathology, and physics. He must study the reactions elicited throughout the entire body before general, wholesale use of a certain drug is warranted. And these reactions are preferably studied on the various systems of lower animals

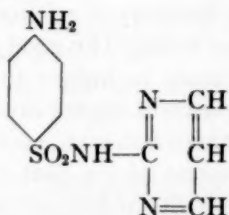
first, rather than run the risk of dangerous symptoms of poisoning developing in man. Let us take, as an example, morphine. Physicians have learned from long experience that morphine will produce in the body several effects depending upon dosage chiefly. They include: alleviation of pain, permitting of sleep, softening and slowing of the pulse, constriction of the pupils in the eye, slowing of respiration, production of constipation, and probably, after long usage, establishment of habit-formation, addiction or physiologic dependence upon morphine. The alert physician must be aware of all of these actions when employing such a potent agent. Because morphine possesses certain undesirable features, the pharmacologists have long attempted to find an adequate substitute for morphine with the hope that virtues might be maintained and detrimental features eradicated. As substitutes, codeine, dilaudid, and other chemicals have been offered. All of these have their weakness still, as well as some advantages over morphine. Still more recently a new substance has been offered to the medical profession under the name of Demerol. This was of major interest because of the threatened shortage of our morphine stock in the United States at the time of Japanese conquest of large Pacific areas including China and some contiguous territory, the natural sources of supply for some of our opium. Much of it came also from Persia, but that country too was threatened by Axis domination. Hence, the outlook for the relief of pain, especially in our armed forces was not too bright since it was estimated that at the time of the Pearl Harbor disaster our morphine supply might last for approximately little less than two years at best. Fortunately for all of us, this outlook was sufficient stimulus to our scientists to seek intensively an adequate substitute for morphine, and preferably one which could be made from our own raw materials, if possible. Amongst the fruits of these researches we find Demerol. But the story of its development was one of slow, careful progress. Following its synthetic production, its toxicity had to be studied in lower animals such as white mice and rats, guinea pigs, rabbits and dogs. Then its effects had to be studied in relation to the blood, bone marrow, vital organs such as the heart, brain, liver and kidneys. We had to learn about its action on the gastrointestinal tract—did it lead to constipation or interfere with digestion? Did it elevate or decrease blood pressure? Was it harmful to the kidneys or reproductive organs? Did it produce liver or brain damage? Might it cause severe jaundice or

lead to stupefaction? In other words, might it be used safely in man or would it probably lead to habit-formation? All of these questions required an answer before the drug was to be introduced to clinical practice. Today we have in Demerol a pain-killing agent which is not the equal of morphine but which very nearly approximates it. It possesses the important advantages of being capable of synthesis and being less habit forming than morphine. And to this achievement, the various sciences contributed. As a teacher of science you can interdigitate your science with that of others for the benefit of your students and welfare of society.

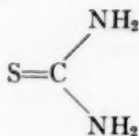
Other illustrations are of equal interest, in particular, the sulfa drugs. From a chemical point of view the teacher of chemistry has an excellent opportunity to attract the interest of his students. Beginning with oxygen he can build progressively, and quickly, through union with hydrogen to form water or hydroxyl groups; added to C in the form of the benzene ring he confronts the student with carbolic acid or phenol, a common antiseptic and locally anesthetizing agent. The removal of the OH radicle and in its place the addition of  $\text{NH}_2$  (not far removed from  $\text{NH}_3$  or ammonia) produces aniline, the basis of many of our staining dyes and also a constituent of some shoe polishes. Of greater importance is the fact that aniline is readily transformed, by proper addition of another  $\text{NH}_2$  group linked with sulphur dioxide ( $\text{SO}_2$ ), a refrigerating gas, to one of the greatest lifesavers offered to mankind, sulfanilamide. Even here the chemist has exercised his ingenuity to modify sulfanilamide to sulfathiazole or sulfadiazine, both of which present certain improvements over their mother substance or precursor. Greater potency with less undesirable side-effects are the goals of all pharmacologists and these have been attained in the sulfa-drugs. Are they fascinating? They are to the physician who theorizes about how these drugs cure infections. He may never learn the proper answer here but how he appreciates their healing properties! If they are fascinating to the physician, how much more so should they be to the chemist and bacteriologist who seek and usually succeed in attaining the improved goals. Thus the process from simple paving bricks, such as the elements, C, O, H, S, et cetera, to the finished highway of powerful chemical lifesavers should prove to be intriguing to any mortal man, if properly presented in all of its dramatic potentialities. Yours for scientific interdigitation.

Would you have another illustration? No, not penicillin, whose story is ever thrilling. Important as this marvelous drug is, I choose for my last illustration a drug known as Thiouracil. The potentialities of clinical value of this chemical were only recently unearthed and that by a rather strange devious route. It was observed by McKenzie, Astwood and others that those

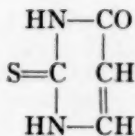
sulfa drugs which possessed an area,  $O=C \begin{matrix} \nearrow NH_2 \\ \searrow NH_2 \end{matrix}$  nucleus, such as found in sulfadiazine,



produced a strange effect in the thyroid gland when fed experimentally to white rats. The thyroid gland increased moderately in size and when looked at through the microscope the thyroid tissue appeared to be different from normal thyroid tissue. It showed signs of suppression of activity. Other urea containing substances were likewise tried. Amongst them were Thiourea and Thiouracil:



THIOUREA



THIOURACIL

Interestingly enough the same effect was observed after feeding these substances to animals. Later, thiouracil was tried in man since it was observed that under its influence the organism had a lower rate of metabolism, promising thus to be of value in the treatment of patients with hyperthyroidism or "toxic goitre" since in this condition a high rate of metabolism prevails. The effects of the drug were extremely fascinating and gratifying. Under its influence it was observed that the basal metabolic rate decreased and signs and symptoms of the disease gradually subsided. In many clinics throughout this country thiouracil

is now being employed to prepare certain goitrous patients for surgical removal of the thyroid gland. What its future may be is problematic but it may not be too much to hope that certain patients suffering from hyperthyroidism may eventually be spared surgery in this condition. Only the future can give us the final answer and we await it with interest. Suffice to emphasize for our purposes today—chemistry and physics as well as physiology and anatomy combined in this instance again to give us, after adequate experimentation, Thiouracil, a chemical for the treatment of a common ailment of mankind.

To recapitulate—as teachers of science, new approaches are open to us in selling our wares. The good teacher is a salesman; he employs every legitimate technique to present his case. And frequently he has the pleasant experience of realizing that some of his seeds have germinated in fertile soil. As a gardener carefully observes the progress of his first favorite bulb, so the teacher anticipates the fruits of his endeavors. By scientifically interdigitating, it may be his just reward to find at harvest not a dry, wrinkled russet but numerous variegated Delicious, Stayman, and McIntoshes. They are his for the asking. To implant is achievement but to inspire is infinite.

---

#### NEW LEAF-COLOR CHART HELPS ORCHARDISTS TO GET REDDER APPLES

The redder the apples the better they sell. But the right shade of green in the apple-tree's leaves is an indicator of how red the apples will be, since healthy dark-green in leaves and lively red in apple skins both result, in part, from proper adjustment in the amount of nitrogen fertilizer fed to the tree.

Working on this principle, two Cornell University faculty members, O. C. Compton and Prof. Damon Boynton, made careful laboratory studies of the color of leaves collected in midsummer from trees under different nutritional conditions. Using their spectrophotometer data a New York City research corporation made up a set of seven carefully compounded printing inks, with which a chart of seven leaf-green shades has been prepared. Now all an apple-grower needs to do is hold the chart alongside sample leaves from his trees, and he will get an idea of how things stand with their nitrogen nutrition.

In general, high nitrogen produces apples of large size but poor color; and since color is the deciding sales factor a compromise must be sought between color and size.

The work of the two Cornell researchers has thus far been confined to one apple variety, the McIntosh. However, since about half the apples raised in New York are of this variety, their studies are considered of particular importance for this state.

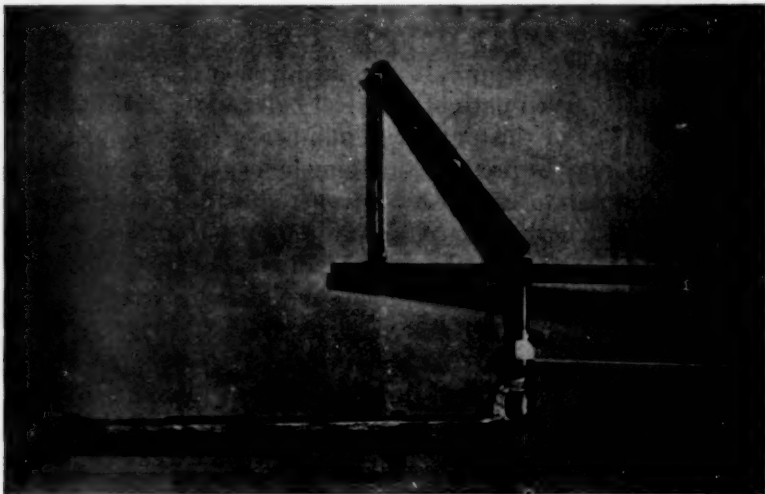
## AN EXPERIMENT ON THE BANKING OF CURVES

WALTER V. BURG

*University of Toledo, Toledo, Ohio*

An elementary study of rotary motion usually includes a brief discussion of the banking of curves. However, in spite of the practical interest of this topic, its treatment is in general limited to a theoretical derivation of the equation for the angle of bank.

In order to enhance the study of this subject by an experimental illustration, the author has developed a simple apparatus, which can be assembled with materials available in every physics department.



A steel pipe of approximately 13 inches length and  $\frac{3}{4}$  inch internal diameter is cut open parallel to its longitudinal axis so as to provide a visible path for the rotating object, a steel ball, whose diameter is somewhat smaller than that of the pipe. In order to confine the ball within the pipe, the latter is closed by two metal plugs or some other kind of obstruction. One end of the pipe is mounted on a hinge, which in turn is screwed to a wooden board of rectangular or circular shape. The board, which is provided with a perpendicular axis passing through its center, carries a counterweight located opposite the pipe. The hinge is

fastened on the board somewhat off center so that on rotating the pipe describes a conical surface having its vertex at the center of the board. By means of the hinge, the angle between pipe and board can be varied over a range of approximately  $45^\circ$ , while a bracket and clamp permit the pipe to be fixed at the desired angle.

The rotary motion of the apparatus may be supplied by a number of different devices. However, since a hand driven rotator can be found in most physics stockrooms, adaptation of such an instrument for the purpose of this experiment will be described briefly. After removing the handwheel and belt, a string several yards long is attached to the set screw of the cylindrical rotator head. Part of the string is then wound around the rotator head, while the remainder is carried over a pulley, set in the same horizontal plane, and a second pulley, placed vertically above the first, to a weight, which is suspended in an elevated position.\*

The apparatus requires some arrangement by which the radius of the curved path of the steel ball can be varied within certain limits. For this purpose a number of rods, about  $\frac{1}{4}$  inch in diameter, may be prepared, ranging in length from one to five inches. In order to establish a given radius of curvature, the rod of proper length is placed into the pipe and fastened to the lower end of the latter. The steel ball, supported by the free end of the rod, thus obtains the desired distance from the axis of rotation.

If the weight is now released from its elevated position, it will produce an accelerated rotary motion of the pipe and the steel ball. During this process of acceleration the centripetal force, required to keep the ball on its circular path, increases steadily until it finally exceeds the force actually supplied by the weight of the ball and the thrust of the pipe. When this condition is reached, the ball will leave its position, thereby producing a clearly audible effect.

The experiment is carried out as follows: After adjusting the positions of pipe and steel ball so as to suit the chosen values for the angle of bank and the radius of curvature, the weight is released. Both the distance and time of its descent are determined, using the aforementioned effect as the endpoint of the observation. From the data thus obtained and the known mag-

\* A few trials will show which weight should be selected for a satisfactory performance of the experiment.

nitude of the radius of the rotator head, the angular velocity of the steel ball at the instant of its departure from the circle can be computed.

In order to show the relationship between the angle of bank  $\theta$ , the angular velocity  $\omega$ , and the radius of curvature  $r$ , a number of experiments are performed, employing different values for  $\theta$  and  $r$ . Any two of these experiments are then compared by forming the ratios  $\tan \theta_1 / \tan \theta_2$  and  $\omega_1^2 r_1 / \omega_2^2 r_2$ . If the determinations are carried out with the proper care, a satisfactory illustration of the desired relationship will be obtained.

---

### TWO ENGINES IN HEAVY TRUCKS RECOMMENDED

Installation of two engines in heavy trucks to give extra power was recommended at the meeting of the Society of Automotive Engineers by Ralph M. Werner, United Parcel Service. Tests during the past 15 years indicate that dual-engined vehicles are efficient and economical, he said. The two engines may operate simultaneously at all times or one may be used only when extra power is required in hill climbing or in mud and snow.

The use of multiple power plants in trucks is by no means a new idea, he explained. It has been tried both in Europe and in America. A truck built in the United States in 1930 was a three-axle unit equipped with two 135-horsepower eight-cylinder engines. Others were built later. In one, which Mr. Werner himself designed, one engine was placed at the rear of the rear-most axle and the other just ahead of the first driving axle.

Costs of building and operating a truck equipped with a single 200-horsepower engine and one with two 100-horsepower engines were contrasted by the speaker. For the two-engine installation the cost will be only 30% of the cost of the single engine with twice the power, he said. When transmissions, clutches, and extra axles are taken into consideration there is still a saving.

"It is conceivable," Mr. Werner concluded, "that on a complete installation of this kind the saving might well run as high as 50%."

---

### LITTLE, BROWN AND D. C. HEATH AND COMPANY MAKE RECIPROCAL ARRANGEMENT

Little, Brown & Company announces that it has recently decided to discontinue publication of school and college textbooks, including Atlantic Monthly Press textbooks, and has sold to D. C. Heath and Company of Boston its active textbook list.

Plans are under discussion between D. C. Heath and Company, Little, Brown & Company, and the Atlantic Monthly Press whereby textbooks issued by D. C. Heath and Company, which have trade edition possibilities, will be handled in trade editions by Little, Brown & Company; trade books or manuscripts originating at Little, Brown & Company and the Atlantic Monthly Press, which have textbook possibilities, will be handled in textbook editions by D. C. Heath and Company.

## MINIATURE DOUBLE STARS

E. M. TINGLEY

*221 North Cuyler, Oak Park, Illinois*

There are several thousand catalogued doubled stars in the heavens. These are pairs of suns. Pairs revolve in their own plane fixed in relation to neighbor stars. Each component of a pair revolves in its orbit around their common center of gravity.

To assist in comprehending the laws governing double stars we will bring, in our imagination, miniature double stars within our range of vision. This is done with the aid of simple arithmetic and the accompanying log log chart.

The elements of performance include quantity of matter or mass in each component, distance between their centers of gravity or size of orbit, and the time required for one revolution.

According to our present philosophy and physics there are two forces that by their equality or balance maintain the orbital diameter of double stars. The attraction of gravity tends to bring the components together and the centrifugal force of revolution in the orbit tends to separate them. These actions extend from microscopic to astronomic distances.

To simplify the picture of our miniature double stars we will consider only pairs having equal components and circular orbits.

In text books on physics these required formulas will be found:

Gravity force between components varies directly as the square of the mass in a component and inversely as the square of the orbit diameter.

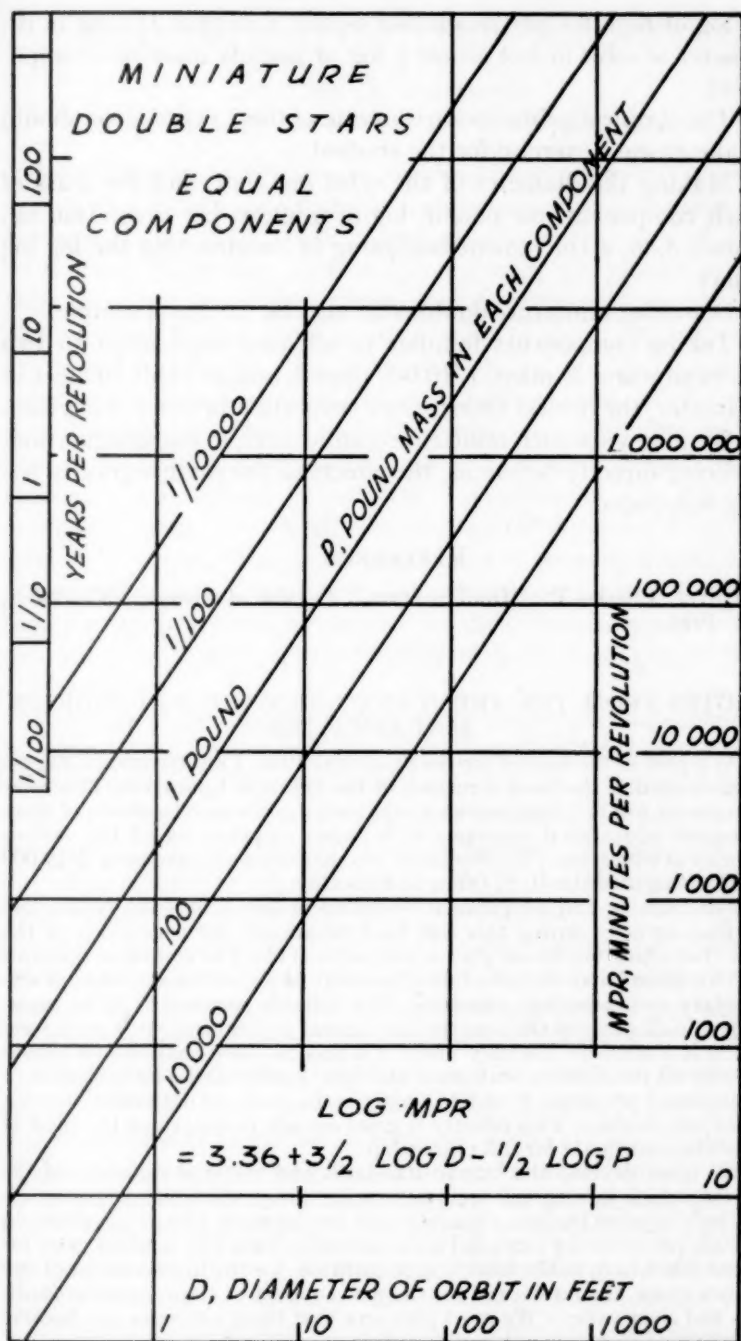
Centrifugal force varies directly as the mass in a component, the diameter of the orbit, and the square of the revolutions per minute.

As the forces balance or are eliminated we may write that the square of the minutes per revolution varies directly as the cube of the orbital diameter and inversely as the mass in a component. These are the bones or the skeleton of the formula.

Inserting numerical values, the formula becomes:

The square of the minutes per revolution equals 5,150,000 times the cube of the diameter of the orbit in feet divided by the pounds mass in a component.

Expressed in logarithms for constructing the accompanying log log chart:



log of minutes per revolution equals 3.36 plus  $3/2$  log of diameter of orbit in feet minus  $1/2$  log of pounds mass in a component.

The simple algebra and arithmetic of these expressions should make an easy exercise for the student.

Making the diameter of the orbit one foot, and the mass of each component one pound, log of minutes per revolution becomes 3.36, a convenient beginning in constructing the log log chart.

One other miniature double star may be cited as a sample.

Taking components familiar to all, each equivalent to two drops of water in mass,  $1/10,000$  pound, and an orbit 100 feet in diameter, the orbital time for one revolution becomes 435 years.

We may view such miniature double stars, in our imagination, as being directly before us, the effects of the earth's gravity being suspended.

#### REFERENCE

- Oliver, Charles P., "Double Stars," *Popular Astronomy*, November, 1944.

---

#### NOTES FROM THE PRESIDENT'S MESSAGE TO CONGRESS JANUARY 9, 1945

As a part of the budget for the fiscal year 1946, I am recommending reorganization of the basic structure of the Office of Education. This reorganization will facilitate service to the states in the developments of more adequate educational programs with proper emphasis on all the various aspects of education. [The President recommended an increase of \$619,000 in the budget of the U. S. Office of Education.]

The training and educational programs of the Army, the Navy, and civilian agencies during this war have broadened our conception of the role that education should play in our national life. The records of selective service reveal that we have fallen far short of a suitable standard of elementary and secondary education. If a suitable standard is to be maintained in all parts of the country, the federal government must render aid where it is needed—but only where it is needed. Such financial aid should involve no interference with state and local control and administration of educational programs. It should simply make good our national obligation to all our children. This country is great enough to guarantee the right of education adequate for full citizenship.

We must develop the human standards and material resources of the nation, which in turn will tend to increase our productivity and most effectively support business expansion and employment. Our program should include provision for extended social security, including medical care; for better education, public health, and nutrition; for the improvement of our homes, cities, and farms; and for the development of transportation facilities and river valleys. We must plan now that these programs can become effective when manpower and materials are available.

## ILLUSTRATING THE CONIC SECTIONS

WALTER H. CARNAHAN

*Purdue University, Lafayette, Indiana*

Many important formulas in physics, aeronautics, engineering and other sciences are quadratic in two variables, and many of the equations encountered in elementary algebra are of the same kind. Neglecting certain special cases, the graphs of all such equations are conic sections, that is, circles, ellipses, parabolas and hyperbolas. From another point of view, these curves are familiar to us all. The earth and all the planets, periodic comets, satellites and other heavenly bodies travel on ellipses. A ball tossed into the air follows a parabola. The curve to be seen in the finished suspension bridge is a parabola. Search light reflectors have surfaces in the form of paraboloids. The shadow or a coin or ring is ordinarily an ellipse. The hyperbola can be used to locate an enemy gun which cannot be seen. Circles are in the wheels on which we ride, in the buttons on our clothes, in the coins we pass over the counter, in the dishes from which we eat. We cannot turn anywhere without seeing examples of the conic sections.

Teachers of mathematics need some simple way of showing how the conic sections are formed and how they differ from each other. There are dissected cones made for this purpose if one happens to have one at hand when it is needed. One can make a cone out of a piece of paper. There is a brand of tooth powder put up in a cardboard cone which can be cut with a kitchen knife to show the sections. A flash light will make the sections on the wall. And, of course, equations can be graphed and studied analytically. There is a simple and attractive way to show these curves that is not often used. Get an ordinary Erlenmeyer flask, fill it a third full of radiator alcohol, cherry juice or your favorite beverage, put in the stopper and you are all set. Place the flask on the table and the surface is a circle. Tilt the flask at any angle and the surface is an ellipse. (Exception: There are two positions of the tilted flask for which the surface is not an ellipse. We note these presently.) As the flask is tilted at various angles, the ellipse changes shape. Technically we say its eccentricity varies, and here is a simple way to get across the meaning of eccentricity. When the flask is tilted so that its axis is parallel to the top of the table, the surface is an hyperbola. When the flask is placed

on its side on the table, the surface is a parabola because the surface is now parallel to an element of the cone. The bottle can be kept on the table or on a shelf in the classroom as a constant reminder to students of just how common the conic sections really are. If the liquid is a nice red or other attractive color, the invitation to look at the sections is much more pleasantly given. A good glass blower can heat the top of the bottle and draw it out to a point, thus completing the cone. Best have the cone about closed up before the liquid is added, especially if the liquid happens to be radiator alcohol. Warning: In making the final seal, be careful not to blow the flame into the flask if alcohol is used. It takes little imagination to picture the results.

---

#### PLANNING SCHOOLS FOR TOMORROW

Two new leaflets in the U. S. Office of Education Series, *Planning Schools for Tomorrow*, are just off the press, according to a recent announcement by John W. Studebaker, U. S. Commissioner of Education.

The series is being prepared to assist both lay and professional groups interested in planning to improve the educational opportunities offered children in certain specific fields. While addressed particularly to local and State planning groups, these leaflets are of wide general interest since the provisions with which they deal are designed not only to meet current educational needs but also to lay a foundation for adequate post-war programs affecting all children.

*Needs of Exceptional Children*, leaflet No. 74, is concerned with the education of children having special physical, mental, or emotional needs. Particularly directed toward the problems of localities in which the education of exceptional children is not yet very far advanced, it defines the problem, explains its importance, and answers such questions as: What special educational services do exceptional children need? To what extent are such services now being provided? What State legislative action should be taken to provide these services? What do the needed services cost? How can the State's total program for exceptional children be unified?

*Pupil Personnel Services for All Children*, leaflet No. 72, will be of interest to parents as well as school officials responsible for any type of school system, since it emphasizes the need of *all* children for the services indicated. It presents the now generally accepted point of view that the pupil as a whole is the important consideration in education, rather than his intellectual capacities and achievements without regard to other aspects of his personality and development. It discusses the need for pupil personnel services and describes such services as they are now provided in efficient school systems. The cost of such services, as well as the type of personnel needed to maintain them adequately, is described on the basis of experience in representative school systems.

Both of these leaflets may be purchased from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., at 10 cents each.

## PANDIAGONAL MAGIC SQUARES ON SQUARE BASES AND THEIR TRANSFORMATIONS

HIRAM B. LOOMIS

*6119 S. Rockwell St., Chicago, Illinois*

In this article we shall examine the structure of pandiagonal magic squares on square bases, using the  $9 \times 9$  square for illustration. This structure is best seen when we use the number system whose base is 9, because we can then consider the units digits separately from the second place digits. We shall divide these pandiagonals into three classes:

Class I. Both sets of digits are built on a plan that applies to any base;

Class II. Both sets are built on a plan that applies only to a square base; and

Class III. The units' digits are built on one plan and the second place digits on the other.

We shall proceed as follows:

- I. Give a method of constructing magic squares that applies to any base, using the  $5 \times 5$  square as an illustration;
- II. Give a method that applies only when the base is a square;
- III. Show how to combine the two methods, using one for units' digits, the other for second place digits; and
- IV. Point out essential differences between the three classes.

### METHOD FOR ANY BASE

Figure 1 is a natural order  $5 \times 5$  square. Our objective is to transform it into magic squares. As we shall use this transformation throughout this article, we give it in detail. In getting figure 2, we make one column (in this case the middle column) the same as in figure 1. We next make the main diagonal of figure 1, i.e., 1, 7, 13, 19, 25, the middle row of figure 2. We then fill in the columns, keeping the numbers in the same cyclic order as in figure 1. Figure 3 is derived from figure 2 in essentially the same way, except that we make the middle row the same and work horizontally instead of vertically. Figure 3 is semi-pandiagonal. Figures 4 and 5 are derived from figures 3 and 4 by similar horizontal transformations. Figure 4 is fully pandiagonal, and figure 5 semi-pandiagonal for the other set of diagonals. Figure 6, which is not pandiagonal at all, may be obtained from figure 2 by a similar transformation, working

with diagonals instead rows and columns. We have every type of  $5 \times 5$  magic squares all related to each other.

FIGURE 1

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

FIGURE 2

16	22	3	9	15
21	2	8	14	20
1	7	13	19	25
6	12	18	24	5
11	17	23	4	10

FIGURE 3

9	15	16	22	3
20	21	2	8	14
1	7	13	19	25
12	18	24	5	6
23	4	10	11	17

FIGURE 4

22	3	9	15	16
14	20	21	2	8
1	7	13	19	25
18	24	5	6	12
10	11	17	23	4

FIGURE 5

15	16	22	3	9
8	14	20	21	2
1	7	13	19	25
24	5	6	12	18
17	23	4	10	11

FIGURE 6

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

This method will work, no matter in what order we arrange the columns of figure 1, or in what order we arrange the rows of the square thus obtained. Everything stated so far will hold for the natural order square on any prime base. If the base is composite, except cases in which 2 enters once and only once as a factor, the method will work for certain definite arrangements of the rows and columns of the natural order square. The method is general.

#### FROST'S METHOD OF WRITING NUMBERS

As the ordinary method of writing numbers on the decimal system is not suited to the separation of units' and second place digits in working with magic squares, we shall use a method devised by the Rev. A. H. Frost, one of the best mathematicians that ever worked with magic squares. About three quarters of a century ago he was a missionary in Nasik, India; worked out a group of magic squares and cubes that he called Nasik; and extended his investigations into hyperspace, actually constructing a model of a Nasik  $7 \times 7 \times 7 \times 7$  for the South Kensington Museum (London). In this writer's opinion, Mr. Frost's method is the method best suited for work in this class of magic squares, though, so far as he knows, no one else has used it.

An illustration will probably best bring out the advantages of his method. Figure 7 is figure 1 written as Mr. Frost might have written it on the number base 5. The main difference is that the number 5 on the base 5 is written 0-5, not 1-0; the number 10 is written 1-5, not 2-0; etc.\*

\* When not using the decimal system, we shall separate units' digits from second place digits by a hyphen. In this way the reader will know that we are using a number base other than 10.

FIGURE 7

0-1	0-2	0-3	0-4	0-5
1-1	1-2	1-3	1-4	1-5
2-1	2-2	2-3	2-4	2-5
3-1	3-2	3-3	3-4	3-5
4-1	4-2	4-3	4-4	4-5

FIGURE 8

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4

FIGURE 9

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

A glance at figures 8 and 9, in which the two sets of digits are separated, should convince a mathematician that the squares could hardly be in better shape for work. The 5 is really needed in the unit's place.

#### 9×9 PANDIAGONAL CONSTRUCTED BY THE GENERAL METHOD

Nine is not a prime. We, therefore, choose from the 144 possible orders for units digits 1, 2, 3, 5, 6, 4, 9, 7, 8; and from the equal number of possible orders for second place digits 0, 1, 2, 5, 3, 4, 7, 8, 6. This gives figure 10 for our fundamental square. Figure 11 is pandiagonal and was derived from figure 10 just as figure 4 was derived from figure 1. If the transformations were kept up with the middle row constant, we would have several fully pandiagonal squares before we reached the square that corresponded to the second semi-pandiagonal square in the 5×5 case.

FIGURE 10

0-1	0-2	0-3	0-5	0-6	0-4	0-9	0-7	0-8
1-1	1-2	1-3	1-5	1-6	1-4	1-9	1-7	1-8
2-1	2-2	2-3	2-5	2-6	2-4	2-9	2-7	2-8
5-1	5-2	5-3	5-5	5-6	5-4	5-9	5-7	5-8
3-1	3-2	3-3	3-5	3-6	3-4	3-9	3-7	3-8
4-1	4-2	4-3	4-5	4-6	4-4	4-9	4-7	4-8
7-1	7-2	7-3	7-5	7-6	7-4	7-9	7-7	7-8
8-1	8-2	8-3	8-5	8-6	8-4	8-9	8-7	8-8
6-1	6-2	6-3	6-5	6-6	6-4	6-9	6-7	6-8

FIGURE 11

7-2	8-3	6-5	0-6	1-4	2-9	5-7	3-8	4-1
0-5	1-6	2-4	5-9	3-7	4-8	7-1	8-2	6-3
5-4	3-9	4-7	7-8	8-1	6-2	0-3	1-5	2-6
7-7	8-8	6-1	0-2	1-3	2-5	5-6	3-4	4-9
0-1	1-2	2-3	5-5	3-6	4-4	7-9	8-7	6-8
5-3	3-5	4-6	7-4	8-9	6-7	0-8	1-1	2-2
7-6	8-4	6-9	0-7	1-8	2-1	5-2	3-3	4-5
0-9	1-7	2-8	5-1	3-2	4-3	7-5	8-6	6-4
5-8	3-1	4-2	7-3	8-5	6-6	0-4	1-9	2-7

We postpone further study of this square until we have the others with which to compare it.

## METHOD APPLICABLE ONLY TO SQUARE BASES

The writer first met a square of this kind in Professor A. L. Candy's book on Pandiagonal Magic Squares. Professor Candy afterwards furnished him other examples by correspondence. He rewrote the squares on the number base 9, separated the digits and by transformations arrived at the characteristic square for units' digits shown in figure 12. The corresponding square for second place digits simply replaces the digits 1 to 9 by 0 to 8.

FIGURE 12

1	1	1	2	2	2	3	3	3
1	1	1	2	2	2	3	3	3
1	1	1	2	2	2	3	3	3
4	4	4	5	5	5	6	6	6
4	4	4	5	5	5	6	6	6
4	4	4	5	5	5	6	6	6
7	7	7	8	8	8	9	9	9
7	7	7	8	8	8	9	9	9
7	7	7	8	8	8	9	9	9

These squares possess two characteristics of the natural order square on a prime base and of any square on a composite base constructed as figure 10 was constructed. The sum along all the diagonals is the same, 45 for the units' square, and 36 for the other. Moreover it is the same for the sum of any consecutive nine numbers met by starting with any number in the square and going along a path whose steps are A cells to the right and B cells down. The differences are in the sums of rows and columns. In the natural order squares no two rows have the same sum, and no two columns have the same sum; but in these squares both rows and columns are in groups of threes with the same sum. These differences show in the resulting magic squares.

Figures 13, 14, 15, and 16 give transformations of these two squares in such form that a units' digit square may be united with a second place digit square so that every unit's digit meets a second place digit once and only once, thus giving every number from 1 to 81 with no repetitions. In figure 17 figures 13 and 16 have been so united. Can we treat this square somewhat as we treated the  $5 \times 5$  natural order square and get a pandiagonal square? Figure 18 is a transformation of figure 17, made by keeping a column constant twice and then keeping a row constant twice. It is pandiagonal, one of the possible pandiagonals from figure 17.

But figures 13, 14, 15 and 16 may be modified and thus yield many variations. The rows of figure 13 are marked A, B, and C. The A's may be interchanged in any way, so may the B's and C's. The columns may be treated in the same way. Moreover three rows or three columns may be taken from one side and placed on the other. Nor does this exhaust the possibilities.

FIGURE 13

1	1	1	2	2	2	3	3	3	A
2	2	2	3	3	3	1	1	1	B
3	3	3	1	1	1	2	2	2	C
4	4	4	5	5	5	6	6	6	A
5	5	5	6	6	6	4	4	4	B
6	6	6	4	4	4	5	5	5	C
7	7	7	8	8	8	9	9	9	A
8	8	8	9	9	9	7	7	7	A
9	9	9	7	7	7	8	8	8	C

FIGURE 14

0	0	0	1	1	1	2	2	2
1	1	1	2	2	2	0	0	0
2	2	2	0	0	0	1	1	1
3	3	3	4	4	4	5	5	5
4	4	4	5	5	5	3	3	3
5	5	5	3	3	3	4	4	4
6	6	6	7	7	7	8	8	8
7	7	7	8	8	8	6	6	6
8	8	8	6	6	6	7	7	7

FIGURE 15

1	4	7	2	5	8	3	6	9
1	4	7	2	5	8	3	6	9
1	4	7	2	5	8	3	6	9
4	7	1	5	8	2	6	9	3
4	7	1	5	8	2	6	9	3
4	7	1	5	8	2	6	9	3
7	1	4	8	2	5	9	3	6
7	1	4	8	2	5	9	3	6
7	1	4	8	2	5	9	3	6

FIGURE 16

0	3	6	1	4	7	2	5	8
0	3	6	1	4	7	2	5	8
0	3	6	1	4	7	2	5	8
3	6	0	4	7	1	5	8	2
3	6	0	4	7	1	5	8	2
3	6	0	4	7	1	5	8	2
6	0	3	7	1	4	8	2	5
6	0	3	7	1	4	8	2	5
6	0	3	7	1	4	8	2	5

FIGURE 17

0-1	3-1	6-1	1-2	4-2	7-2	2-3	5-3	8-3
0-2	3-2	6-2	1-3	4-3	7-3	2-1	5-1	8-1
0-3	3-3	6-3	1-1	4-1	7-1	2-2	5-2	8-2
3-4	6-4	0-4	4-5	7-5	1-5	5-6	8-6	2-6
3-5	6-5	0-5	4-6	7-6	1-6	5-4	8-4	2-4
3-6	6-6	0-6	4-4	7-4	1-4	5-5	8-5	2-5
6-7	0-7	3-7	7-8	1-8	4-8	8-9	2-9	5-9
6-8	0-8	3-8	7-9	1-9	4-9	8-7	2-7	5-7
6-9	0-9	3-9	7-7	1-7	4-7	8-8	2-8	5-8

FIGURE 18

0-1	3-3	0-5	7-8	1-7	7-3	5-6	8-5	5-7
0-6	7-9	4-2	7-1	5-4	2-9	5-8	0-2	6-4
4-3	1-5	5-5	2-7	8-3	0-3	6-5	3-7	7-7
8-9	2-8	8-1	3-4	6-6	3-8	1-2	4-1	1-6
8-2	3-5	0-7	3-9	1-3	7-5	1-4	8-7	5-3
0-8	6-1	1-1	7-6	4-8	8-8	5-1	2-6	3-6
4-5	7-4	4-9	2-3	5-2	2-4	6-7	0-9	6-2
4-7	2-1	8-6	2-5	6-8	3-1	6-3	4-6	1-8
8-4	5-9	6-9	3-2	0-4	4-4	1-9	7-2	2-2

## COMBINED METHOD

In figure 19 the units' digits of figure 10 are combined with the second place digits of figure 17. It should be noted that the arrangement is such that every unit's digit meets every second-place digit once and only once. One transformation of figure 19 keeping a column constant followed by two keeping a row constant gives figure 20, one of the possible pandiagonals that can be obtained from figure 19.

FIGURE 19

0-1	0-2	0-3	3-5	3-6	3-4	6-9	6-7	6-8
3-1	3-2	3-3	6-5	6-6	6-4	0-9	0-7	0-8
6-1	6-2	6-3	0-5	0-6	0-4	3-9	3-7	3-8
1-1	1-2	1-3	4-5	4-6	4-4	7-9	7-7	7-8
4-1	4-2	4-3	7-5	7-6	7-4	1-9	1-7	1-8
7-1	7-2	7-3	1-5	1-6	1-4	4-9	4-7	4-8
2-1	2-2	2-3	5-5	5-6	5-4	8-9	8-7	8-8
5-1	5-2	5-3	8-5	8-6	8-4	2-9	2-7	2-8
8-1	8-2	8-3	2-5	2-6	2-4	5-9	5-7	5-8

FIGURE 20

2-2	5-3	2-5	3-6	6-4	3-9	7-7	1-8	7-1
3-5	6-6	0-4	7-9	1-7	4-8	2-1	5-2	8-3
4-4	1-9	4-7	8-8	5-1	8-2	0-3	6-5	0-6
8-7	2-8	8-1	0-2	3-3	0-5	4-6	7-4	4-9
0-1	3-2	6-3	4-5	7-6	1-4	8-9	2-7	5-8
1-3	7-5	1-6	5-4	2-9	5-7	6-8	3-1	6-2
5-6	8-4	5-9	6-7	0-8	6-1	1-2	4-3	1-5
6-9	0-7	3-8	1-1	4-2	7-3	5-5	8-6	2-4
7-8	4-1	7-2	2-3	8-5	2-6	3-4	0-9	3-7

## COMPARISON OF THE SQUARES

The differences between the squares in figures 11, 18, and 20 appear most clearly when they are transformed into magic squares that are not pandiagonal for either set of diagonals. Figures 21, 22, and 23 give these transformations. Two exhibit a peculiarity to which the writer has never seen attention called, a peculiarity due to the fact that the base, 9, is itself a square. In figure 22 there are three diagonals in each set that have a summation of 36 nines plus 45 (369, the critical summation for a  $9 \times 9$  square). In figure 23, this is true of only one set of diagonals. Only the main diagonal of the other set has that summation. In figure 21, only the main diagonals have that summation. The letters *R* and *L* in the figures indicate the diagonals that have this summation. Begin with the number immediately below the letter and follow down and to the right or left as the letter indicates.

FIGURE 21

8-5	0-8	2-6	3-1	7-4	6-2	1-9	5-3	4-7
0-7	2-5	3-8	7-6	6-1	1-4	5-2	4-9	8-3
2-3	3-7	7-5	6-8	1-6	5-1	4-4	8-2	0-9
3-9	7-3	6-7	1-5	5-8	4-6	8-1	0-4	2-2
7-2	6-9	1-3	5-7	4-5	8-8	0-6	2-1	3-4
6-4	1-2	5-9	4-3	8-7	0-5	2-8	3-6	7-1
1-1	5-4	4-2	8-9	0-3	2-7	3-5	7-8	6-6
5-6	4-1	8-4	0-2	2-9	3-3	7-7	6-5	1-8
4-8	8-6	0-1	2-4	3-2	7-9	6-3	1-7	5-5

FIGURE 22

R			L	R			L	RL
6-8	3-1	6-3	4-6	1-8	4-7	2-1	8-6	2-5
7-2	2-2	8-4	5-9	6-9	3-2	0-4	4-4	1-9
3-3	0-5	7-8	1-7	7-3	5-6	8-5	5-7	0-1
5-4	2-9	5-8	0-2	6-4	0-6	7-9	4-2	7-1
3-7	7-7	4-3	1-5	5-5	2-7	8-3	0-3	6-5
2-8	8-1	3-4	6-6	3-8	1-2	4-1	1-6	8-9
1-3	7-5	1-4	8-7	5-3	8-2	3-5	0-7	3-9
2-6	3-6	0-8	6-1	1-1	7-6	4-8	8-8	5-1
7-4	4-9	2-3	5-2	2-4	6-7	0-9	6-2	4-5

FIGURE 23

R	L		L					L
2-5	1-8	3-6	7-1	6-4	2-2	3-9	5-3	7-7
1-7	3-5	4-8	6-6	2-1	0-4	5-2	7-9	8-3
0-3	4-7	6-5	8-8	0-6	5-1	4-4	8-2	1-9
4-9	3-3	8-7	0-5	2-8	4-6	8-1	7-4	0-2
3-2	8-9	6-3	2-7	4-5	5-8	7-6	0-1	1-4
5-4	6-2	2-9	1-3	5-7	7-5	6-8	1-6	3-1
6-1	8-4	1-2	5-9	4-3	6-7	1-5	0-8	5-6
8-6	1-1	2-4	4-2	6-9	7-3	0-7	5-5	3-8
7-8	2-6	4-1	3-4	7-2	0-9	2-3	3-7	8-5

The close relation that holds between all the transformations of a given square such as we have been using deserves attention. Perhaps it is best illustrated by the following: Start with any number in figure 18, follow a path whose steps are 4 cells to the right and 1 cell down until you meet nine numbers consecutively. These numbers will lie on one of the nine diagonals of the *R* set of figure 22. If the steps are 2 cells right and 1 down, you get one of the other diagonals; if 3 cells right and 1 down, you get one of the columns; etc. In a  $9 \times 9$  square there are six independent sets of crooked paths in addition to rows, columns and diagonals making in all ten sets of nine paths each. Every one of the 81 numbers is in one and only one path in each of the ten groups, and no two paths have more than one member in common. Yet in all the transformations considered in this article, though a

group of numbers that is now a row may become a column, a diagonal, or along any crooked path, the group will maintain its identity throughout; and its brother rows will also maintain their identities and also be brothers in the new set of paths. This stick-to-it-avness is a property of more than a group of magic squares; it is a property of any and every square array, even of a Greek phalanx of soldiers, a property that has its extension to any and every cubic array, and to any and every corresponding array in hyperspace.

But this is beyond the scope of this article. Apart from the mere mechanics of the construction, the points to be emphasized here are: The "oneness" of such a set of transformations as the three given in this article, each being but one perspective of a single whole, a whole that includes more than mere magic squares: The possibility of getting insight into the structure of square arrays by treating numerical squares on the number base that is the base of the square: and The advantage of an appropriate way of writing numbers on that base.

---

#### AIR CARGO OF RIFLE BARRELS DEFLECTS PLANE'S COMPASS

The near crash of an Army Air Transport Command plane loaded with a ton and a half of rifle barrels for troops leaving for Attu has resulted in a new air express regulation. Today, when anything capable of influencing the magnetic field of a compass is destined to be flown by air, the words "Magnetic Material" must be plainly marked on the shipment, indicating that it requires special treatment.

Flying in Alaska with the cargo of rifle barrels, two United Air Lines pilots assigned to ATC operations discovered that their plane and Mount McKinley were about to have an argument. By skillful flying they avoided the craggy peak. Realizing that their compass must be off, the two flyers investigated and found that the quantity of metal in the rifle barrels had attracted the normal magnetic lines of force. This change in the magnetic field deflected the plane's compass approximately 20 degrees.

Special procedures have been set up because products containing iron might deflect the plane's compass from the correct reading. All large quantities of magnetic materials must be carried in the rear cargo compartment, as far away from the plane's compass as possible.

Magnetic materials acceptable as air cargo include only those items containing magnets with fields not confined. Included on the list are ammeters, galvanometers, magnetos, permanent magnets, motors or generators incorporating permanent magnets, photo-electric light meters, thermacouple meters, voltmeters and ferrous materials.

Magnetic materials carried in the plane's cockpit and front cargo compartment must be demagnetized before being loaded aboard the plane. This procedure has been established for a long time in airline maintenance.

## BOOK CLUBS FOR PROFESSIONAL READING

J. R. MAYOR

*Southern Illinois Normal University, Carbondale*

Each year in every teaching field, even during the war, there have been a number of books published that a teacher should know about and should at least partially read. With numerous calls for the time and money of the teacher, however, this important "keeping up" process is often slighted. One means of meeting the difficulty, that has been used successfully by a group of mathematics teachers, is described in this paper. The plan should work as well for other fields.

The teachers maintain a book club as one activity of their regional organization which is affiliated with the area division of the state education association and with the National Council of Teachers of Mathematics.<sup>1</sup> The membership list of the club is revised each year. The dues are one dollar with a refund at the end of the year upon auction of the books. As nearly as possible a reading list is chosen from publications of the past twelve months. Each year the list includes a variety of books of general interest to the mathematics teacher. A Yearbook of the National Council is usually available. It usually has been possible to select one book on history, one on methods, another devoted to recreations, one on applications or related fields, and one which is an expository treatment. A partial list for this year is:

Archibald, Raymond Clare, *Outline of the History of Mathematics*. 5th edition. 1941.

Coleman, Robert Jr., *Development of Informal Geometry*. 1942.

Committee on Higher Mathematics of the Association of Teachers of Mathematics of New York City, *Selected Topics in Higher Mathematics for Teachers*. 1943.

Hanson, Paul P., *Military Applications of Mathematics*. 1944.

Lieber, Hugh Gray, and Lillian R. Lieber, *The Education of T. C. Mils* (The Celebrated Man in the Street). Revised and enlarged edition. 1944.

National Council of Teachers of Mathematics, *Multi-Sensory Aids in the Teaching of Mathematics*. 18th Yearbook of the N.C.T.M. 1945.

*A Treasury of Science*. Edited by Shapley, Rapport, and Wright. 1943.

White, W. F., *A Scrapbook of Elementary Mathematics*. 4th edition. 1942.

The teachers have chosen to have the circulation so arranged that they have each book two weeks, with a two weeks interval between books. This spacing serves two purposes. The cost is

<sup>1</sup> See *Mathematics Teacher*, January, 1941, p. 23.

less and the teacher feels less pressure in planning his work. For illustration, Mr. Brown of Marion will have the 18th Yearbook two weeks from October 30–November 13. He will then mail it to Mr. Smith who teaches in the town nearest Marion. Mr. Smith will hold the Yearbook for two weeks before mailing it on. In the meantime Brown will have no book from November 13–27. He will, however, receive the next book on the list on November 27.

The membership of the club has varied from ten to twenty. If a club had more than twenty members, a different plan of circulation would be desirable. For example, two separate groups might be organized.

The following quotations from teachers are typical of the reactions of members:

"I like the Book Club because one will read the books received. Too often otherwise we get so busy our reading is put off and never done. I have been a member since the club was organized and enjoy the type of books sent. It also gives us a chance to buy some of the books we especially enjoyed."

"I have enjoyed the Book Club chiefly because we may learn about many books without buying them. Also, I do not have available lists of the books such as the ones the Book Club provides. In the third place, I appreciate the cooperation of mathematics teachers in our section of the country in promoting reading to keep up on present day problems."

"Through the several years that I have been a member of the Book Club I have found it very worthwhile, even though I could not spend as much time with the books as I would like to have done."

"Even though I have been shifted to the P. E. Department for the duration, I still enjoy and profit by membership in the Book Club. It helps me keep up with some of the advances and changes made in the field of mathematics."

A list of some of the books, with publishers, which have been on the Book Club lists in the past four years is presented to give a better idea of possibilities:

- Bakst, Aaron, *Mathematics Its Magic and Mystery*. P. Van Nostrand and Co. Inc. 1941.
- Butler, Charles, and Lynnwood Wren, *The Teaching of Secondary Mathematics*. McGraw-Hill Book Co. 1941.
- Courant, Richard, and Herbert Robbins, *What Is Mathematics*. Oxford University Press. 1941.
- Dantzig, Tobias, *Number, The Language of Science*. 3rd edition. The Macmillan Company, New York. 1939.
- Fehr, H. H., *A Study of the Number Concept of Secondary School Mathematics*. Edwards Brothers Publishing Company. 1940.
- Glaser, Edward H., *An Experiment in the Development of Critical Thinking*. Bureau of Publications. 1941.
- Hardy, G. H., *A Mathematician's Apology*. The Macmillan Company. 1940.

- Howard, Homer, *Mathematics Teachers' Views on Certain Issues in the Teaching of Mathematics*. Bureau of Publications. 1941.
- Jones, S. I., *Mathematical Clubs and Recreations*. S. I. Jones Co. 1940.
- Kasner, Edward, and Joseph Newman, *Mathematics and the Imagination*. Simon and Shuster. 1940.
- Lazar, Nathan, *Importance of Certain Concepts, and Laws of Logic for the Study and Teaching of Geometry*. Bureau of Publications. 1938.
- Loomis, Elisha A., *The Pythagorean Proposition*. Edwards Bros. Inc. 1943.
- McKay, Hubert, *Odd Numbers, Or Arithmetic Revisited*. The Macmillan Company. 1940.
- Minnick, J. H., *Teaching Mathematics in the Secondary School*. Prentice-Hall, Inc. 1939.
- Row, T. Sundara, *Geometric Exercises in Paper Folding*. The Open Court Publishing Company. Revised edition. 1941.
- Smith, D. E., and Jekuthiel Ginsburg, *A History of Mathematics in America before 1900*. The Open Court Publishing Company. 1934.
- Steinhaus, H., *Mathematical Snapshots*. G. E. Stechert and Co. 1938.
- Sullivan, J. W. N., *Life of Newton*. Macmillan Company, 1941.
- Yates, R. C., *The Trisection Problem*. The Franklin Press, Inc. 1942.
- Zant and Diamond, *Elementary Mathematical Concepts from the Historical and Logical Point of View*. Burgess Publishing Company. 1941.

---

#### GLASS TRANSMITTING MORE ULTRAVIOLET LIGHT PROMISED

Volume production of aluminum metaphosphate planned for the post-war era will bring into commercial availability a useful glass. From aluminum metaphosphate can be manufactured glass which transmits a substantially greater amount of ultraviolet light such as aids the formation of vitamin D. Aluminum metaphosphate's use in the making of glass represents a radical departure from such bases as lead, well-known to the ancients, and from the more recent combination of sand, soda ash and lime.

Postwar fluorescent lights may utilize phosphate glass due to its ultraviolet permeability, and it also may be found useful in windowpanes of hospitals and solariums.

Because they show less tendency to yield a haze, phosphate glasses are considered likely to solve some optical problems associated with astronomy, photography and related fields.

Phosphate glasses show improved weather resistance, improved color control, improved melting and working characteristics and improved resistance to some acids. Used in insulators, they reduce loss of electrical current as compared with other glass insulators.

A companion metaphosphate has properties that suggest its ultimate use as a heat insulator. Its volume increases 500-fold under high temperatures. Another metaphosphate, also ready for volume production, is used in ceramics.

---

Today and every working day of the year, 17 more American working men have unnecessarily lost the sight of one or both eyes as the result of occupational hazards, according to the National Safety Council.

## DIAGRAMS FOR SPECIFIC HEAT AND CHANGE OF STATE PROBLEMS

H. F. COPE

*Medford Senior High School, Medford, Oregon*

High school students usually find considerable difficulty in solving specific heat and change of state problems. To help them along, the writer has used a system of heat diagrams for the last few years with the result that now students are generally more successful in their efforts than formerly. These diagrams are passed along with the hope that other teachers will find them useful in their physics classes.

In these diagrams, changes of state are represented by horizontal lines while heating and cooling are represented by vertical lines. Students are quick to see that changes of state do not involve temperature changes, but that heating and cooling do represent certain temperature ranges which have to be obtained by subtracting the final temperature from the upper temperature for cooling, and subtracting the lower temperature from the final for heating. Students will also note that all vertical lines call for " $msl$ " terms in the equation, while horizontal lines call for expressions representing changes of state.

Each one of the six diagrams illustrated will be explained briefly, followed by a heat of fusion problem involving the large diagram (f) together with its solution.

Diagram (a). This diagram is used for specific problems in which no mention is made of a calorimeter.

Diagram (b). This is similar to (a) but with calorimeter included. Note that the hot metal is cooled and the cold water and cold calorimeter are heated.

Diagram (c). This diagram calls for a change of state, heating, and another change of state in passing from ice to steam. Note the ice, the ice water heated to 100 deg. C. and the hot water vaporized.

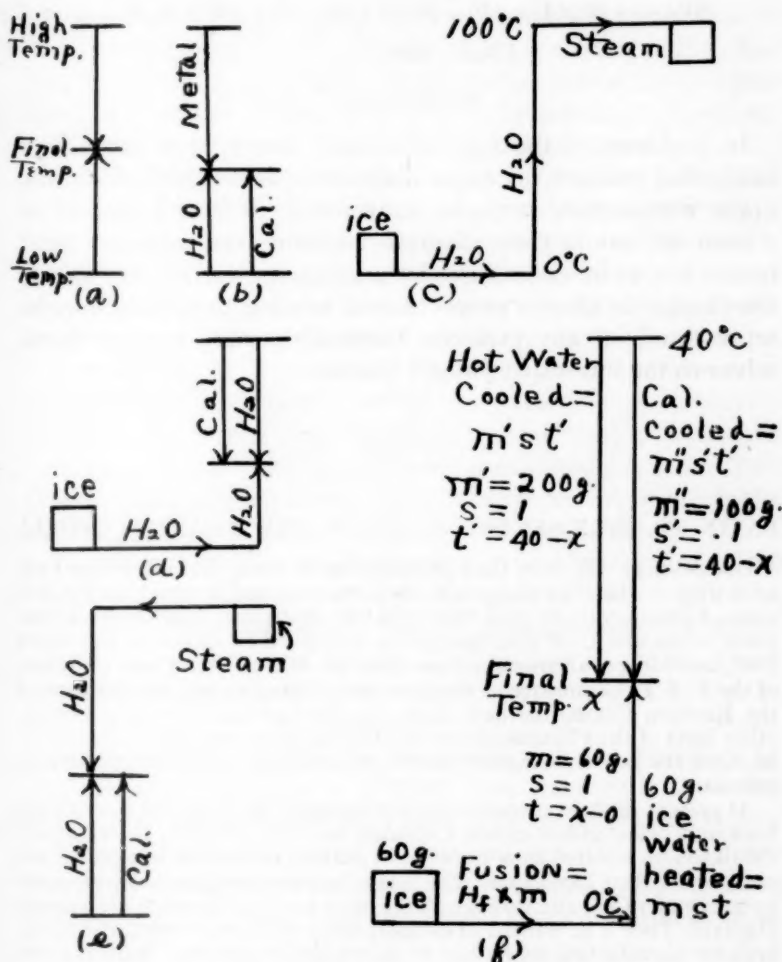
Diagram (d). This represents the ordinary heat of fusion problem. Note the ice, the ice water, the warm water and the warm calorimeter.

Diagram (e). This diagram is used for the ordinary heat of condensation problem. Note the steam, the water from the steam, the cold water and the cold calorimeter.

Diagram (f). This is the large diagram and similar to (d) with values given as they appear in the problem given below.

### PROBLEM-HEAT OF FUSION

Sixty grams of ice at 0 deg. C. are added to 200 grams of water at 40 deg. C. in a copper calorimeter having a specific heat of .1 and weighing 100 grams. (Consider the heat of fusion of ice as 80 cal./g.)



Note: Diagram (f) calls for three vertical or "m s t" lines and one horizontal or change of state line.

Equation:

$$H_f m + m s t = m' s t' + m'' s' t'$$

Evaluate terms

$$H_f = 80 \text{ cal.}; \quad m = 60 \text{ g.}; \quad s = 1; \quad t = (x - 0)$$

$$m' = 200 \text{ g.}; \quad t' = (40 - x); \quad s' = .1; \quad m'' = 100 \text{ g.}$$

Substituting and solving for  $x$

$$80 \times 60 + 60 \times 1(x - 0) = 200 \times 1(40 - x) + 100 \times .1(40 - x)$$

$$270x = 3600$$

$$x = 13.3 \text{ deg. C. Ans.}$$

In problems of the type illustrated above, it is more than likely that without the aid of diagrams students will obtain the upper temperature range by subtracting 40 from  $x$  instead of  $x$  from 40, but in these diagrams students can actually "see" how it has to be done. Also, by making the vertical or " $m s t$ " lines longer or shorter proportionate heating or cooling may be represented for any problem. Other ideas may present themselves to the interested physics teacher.

#### LATIN TO TAKE SECOND PLACE IN NEW PHARMACOPOEIA

Doctors may still write their prescriptions in Latin, but when they look up a drug in the Pharmacopoeia, they want to find it under its English name. Consequently English titles will take first place, Latin titles second place in the new U. S. Pharmacopoeia, scheduled to appear in December 1945, according to an announcement from Dr. E. Fullerton Cook, chairman of the U. S. P. Committee of Revision here. Although medical members of the Revision Committee have been the chief advocates of this change, other users of the Pharmacopoeia besides physicians are expected to benefit, since the new style makes possible the grouping of related products in one place.

At present digitalis preparations, for example, are scattered through the book under class names such as Capsulae, Injectio, Tabellae, Tinctura, and the like. This resulted from the style of putting substances in alphabetical order under their Latin titles. The person looking up digitalis capsules had to hunt under Capsulae through all the other kinds till he came to Capsulae Digitalis. Then if he wanted to compare these with requirements or standards for digitalis tablets, he had to search under Tabellae. With the new style, he will merely look for digitalis. All U. S. P. digitalis preparations will be grouped there in alphabetical order.

The Latin titles will not be dropped, but will be placed after the English titles.

## INDIAN MEDICINE

JOHN S. WRIGHT

*Eli Lilly and Company, Indianapolis, Indiana*

In the last few years Indian medicine has been the subject of study and of publications by both pharmacists and physicians who have endeavored to evaluate it anew in the light of present knowledge. This article is based upon the work of a number of authorities cited in the bibliography. Free use has been made of the texts of several, and special indebtedness is acknowledged to the writings of W. T. Corlett, M.D., a dermatologist, and Eric Stone, M.D., a surgeon. Each of these authorities has produced a book that is helpful as well to laymen as to physicians who are interested in aboriginal American medicine.

Dr. Corlett says that since the usages, customs and knowledge of the medicine men of the widely scattered tribes varied, it is hard to generalize on Indian medicine. However, simple treatments and first aids, as among the whites, were understood by many. In some tribes (the Crows, for example) individuals who cured by means of roots and herbs were known by a term which is best translated as "doctor." The shamans or medicine men who operated additionally as healers were makers of charms and divinations and used "supernatural" methods. These men functioned also as priests and served as spiritual guides for their people. Thus medicine and theology were united in much the same order as with some of the peoples of the Old World.

The sincere medicine man felt that he had been called to the profession just as men in modern days feel called to the ministry. These calls came to the Indians through dreams, extraordinary experiences and visions. Among the Blackfeet sometimes an older medicine man would select a boy of twelve or thirteen to become his understudy and finally his successor. Francis LaFleshe, an Indian connected with the Smithsonian Institution and quoted by Corlett, says that while the office was to some extent hereditary, there were instances well known in his own tribe where men had refused to instruct their own sons in the sacred rites because their character lacked some of the essential requisites. Boys might also apply voluntarily to an established medicine man and if he were accepted would pay for his instruction. In most cases it seems that he was chosen because of some extraordinary quality of mind and body and a keen understanding of human nature.

The course of training was long and arduous, extending in some instances to ten or fifteen years. Part of the training consisted of meditation and solitude, fasting, physical torture, performing feats of physical endurance and training of will power. Magic was frequently included in the course, Periodically he underwent a sort of examination by his preceptor. After finishing the course of seven "tents" the young man was "graduated" in a public ceremony. Thereafter he might act as a lesser medicine-man and advance through death or superannuation of the older members of the fraternity. The younger medicine man found it just as hard to build up a practice as do young white physicians. Some of them were obliged to answer any sick call. Occasionally older ones referred cases they did not want to their pupils who were glad to make contacts in this manner. The young medicine men thus got what then corresponded to our pauper practice, or those able to pay small fees only. If successful in his treatments, he gradually acquired a clientele of better paying patients, for the fees fluctuated with the economic position of the patient.

Many other practices of the medicine men will sound as familiar as the above. When in a dilemma the medicine men called each other in consultation. This was done sometimes to decide on the kind of hocus-pocus to be employed to rid the patient of the effects of a bad dream or witchery. If a young or inexperienced medicine man had two or three deaths it might cost him his life unless he could placate the relatives of the patient. Among the Indians it was bad form to ask a medicine man how he was treating a patient and one who talked too much was not held in high repute. In some tribes (the Algonkians, for example) the shamans were organized into religious secret societies or cults. LaFleshe says that there were also medicine men who corresponded to our modern quacks. These men were shrewd, crafty and unscrupulous. The intelligent classes held them in contempt, while the ignorant feared them. While they were healers of a sort, they were also sleight-of-hand experts, tricksters and given to all manner of unscrupulous practices. The true medicine men, however, were sincere and honest according to their light and sought to do good to their patients. They were truthful, patient, prudent and restrained in speech. They spent much time in meditation and prayer and often fasted. While the shaman was supposed to have great power, it was believed that if he abused this power by using it to punish a personal enemy,

the power would strike back with fatal consequences.

Throughout the continent religion and medicine were linked in the minds of the American Indian as they were in the minds and practices of the Egyptians and other ancient peoples. The medicine man was priest and physician by turns. He was a sort of spiritual leader of the people. Long Lance of the Blackfoot tribe says in speaking of the education and training of the Indian Youth, "Our moral training was entirely in the hands of our mothers. They told us about the Great Spirit, and that when we grew older the Great Spirit would delegate some other good spirit whose home was in the spirit world to be our guide and to look after our well-being. This spirit would give us 'medicine' (which meant some lucky charm), as well as our medicine song and our death song; the former to be sung at all times when in trouble or distress; the latter when we were called on to die."

The medicine man was called in early to perform a sort of christening ceremony. On the eighth day after birth the parents sent for him. In due time he arrived clad in his priestly garb, and raising his right hand to the sky he invoked nature:

"Ho! Ye Sun, Moon, Stars, and all ye that move in the heavens;  
I bid ye hear me!  
Into your midst has come a new life.  
Consent ye, I implore!  
Make its paths smooth, that it may reach the brow of the first hill!"

He then invoked the clouds, rain, mist and all that move in the air to make the path smooth to the brow of the second hill. Next the invocation was to the valleys, rivers, lakes, trees, grasses and all of the earth to make the child's path smooth to the brow of the third hill. Then he invoked the birds and all that fly in the air, the animals, great and small, that dwell in the forest, the insects among the grasses and in the ground to make the path smooth to the brow of the fourth hill. Finally all the heavens and the earth and the air were invoked to make the path smooth when the child travels beyond the four hills. The whole ceremony was one of great solemnity and majesty and genuinely poetic in both spirit and phraseology.

The theory of disease most widespread among the Indians was that some object, such as a fishbone, stone, thorn, stick or evil spirit had entered the body. The treatment, realistic or supernatural, had for its purpose the extraction or expulsion of the intruder. The Mayans practiced a sort of homeopathy, that is, that like cures like. For instance, a skin eruption resembling

the sting of wasps was treated with a poultice made from the nests of wasps and other stinging insects. Vines bearing a certain resemblance to snakes were employed to cure snake bite. Jaundice was cured by decoctions of yellow grubs, seeds or plants, while the vomiting of blood was treated with preparations based on plants or seeds of a red color. While some of these practices sound absurd to us today, Dr. Roy L. Moodie, in the *Annals of Medical History*, states that he believes that no primitive and ancient race of people anywhere in the world had such a field of medical knowledge as had the pre-Columbian Peruvian Indian. A few years ago Dr. Youngken, of the Massachusetts College of Pharmacy, published a list of 75 Indian drugs, of which 57 are now recognized as of sufficient importance as to be retained in the United States Pharmacopoeia or the National Formulary. Among the drugs of importance that have come down to us through the Indians of the Americas are cinchona, coca, cocoa, curare, cascara, aloes, tobacco and tolu.

In 1940 the Johns Hopkins Press published a book on Aztec Indian medicine in translation from a Latin manuscript by Martin de la Cruz in 1552. De la Cruz was a student at the Jesuit College established at Santa Cruz, Mexico, for the sons of the upper classes of natives. He translated this document, known as the *Badianus Manuscript*, from a treatise on drugs by an Aztec physician. It then found its way to the Vatican where it remained unknown except to scholars until about ten years ago. Through the efforts of the Smithsonian Institution it was translated into English and published. This herbal describes and illustrates more than two hundred plants employed in Aztec medical practice. The illustrations are so accurate that the plants are identifiable. The Jesuits and Spaniards regarded the medicine practiced by the Aztecs equal to, if not superior, to the treatments then in vogue in Europe 400 years ago. The *Badianus Manuscript* indicates that the Aztec's achievements in materia medica were worthy of comparison at least with similar works by Europeans of the same period.

While herbs were an important part of Indian medicine, they were attributed with different powers in different tribes. Perhaps this was due in part to the fact that when a medicine man discovered an herb which seemed to cure a certain condition he kept it secret. Some of the remedies were known generally and might have been referred to as "home remedies." Besides the decoctions of herbs for treatment of disease, some of the tribes

practiced bleeding, but various reasons were given for this practice. Sweating and bathing was practiced by other tribes, but these were prescribed for different reasons in the various tribes. Songs were usually an important part of the treatment. Corlett says that with the Blackfoot tribe, at any rate, the treatment that should be administered was revealed to the medicine man through a vision or dream. Anything short of such a revelation was regarded as of no medicinal value.

Perhaps the Indian's principal achievement medically was in the field of Surgery. Dr. Eric Stone says that the average Indian knew infinitely more about the anatomy of the body than the average white man, but the Indian medicine man knew much less than the European physician. He attributes this not so much to a lack of intelligent approach to the subject as to his dependence on Stone Age implements. However, even with these crude instruments it is remarkable what he was able to accomplish. The outstanding operation of the Indians of the Southwest, Mexico, Peru and Central America of pre-Columbian days was the trephining of the skull. The degree of bone repair of the trephined skulls indicates that there was a relatively high survival rate. Skulls have been found showing the frontal sinus had been operated upon to allow drainage, an operation paralleled only in modern surgery. Little is known of their methods in such operations except that they were done without iron or steel instruments.

Dr. Stone says that the most interesting operation performed in the United States was the incision and drainage of the chest in cases of empyema. This was practiced by the Indians of the Great Lakes area who were peculiarly subject to lung infections. The ribs were not resected, but very decent drainage and a high percentage of cures were attained by this method of treatment, in what we are prone to consider the wilds of America and at a time when it was seldom or never employed in Europe.

Boils and abscesses were opened and drained by wicks of shredded bark or reeds. The North Carolina Indians understood the use of ligatures, while other tribes understood the importance of opening and cleaning wounds, of suturing and the difference between healing by first intention and that after supuration. The Northwestern Indians insisted that wounds should heal from the bottom. To accomplish this they placed a thin membrane of bark between the surfaces before placing their stitches. Stone says their technic in this surgical procedure

would be approved today. They used washes, poultices, packs and powders, some of which actually had some antiseptic property. Dr. Stone insists that the Indians were carrying out more rational procedures in the care of wounds than their European contemporaries even of the 15th and 16th centuries. This is also verified by early military observers in America. The Indians were so adept at devising splints that deformities from fractures were rare. They also managed dislocations with unusual success. Apparently major amputations were never attempted.

The Indians had little regard for the white man's cures. They felt that only Indian medicine could cure an Indian. They also believed that the white man brought many diseases among them and that it was done deliberately. A notable exception to this attitude of the Indians was reported in the *Medical Journal and Record* in 1931. This article tells of the acceptance of vaccination as a method of dealing with smallpox. Smallpox was a great scourge to the Indians. In 1807 Dr. Jenner sent the Chief of Five Nations, in care of Col. Wm. Claus, his little book on the "Extermination of Smallpox." So appreciative were the Indians of this assistance that they assembled at Ft. George that fall and sent Dr. Jenner the following address:

"Brother, Our Father has delivered to us the book you sent to instruct us how to use the discovery which the Great Spirit made to you, whereby the smallpox, that fatal enemy of our tribe, may be driven from the earth. We have deposited your book in the hands of a man of skill whom our Great Father employs to attend us when we are sick or wounded. We shall not fail to teach our children the name of Jenner, and to thank the Great Spirit for bestowing upon him so much wisdom and so much benevolence. We send with this a belt and a string of Wampum in token of our acceptance of your precious gift, and we beseech the Great Spirit to take care of you in this world and in the land of spirits."

When we compare this gracious and open-minded acceptance of one of the greatest benefactions ever bestowed on the race with the obstinate and stupid opposition which many whites even to this day, offer to vaccination, we should be slow to make pronouncements regarding the inherent superiority of white mentality.

Dr. Corlett, summarizing his study of the medicine man says, "Had America never been visited by the European we can well imagine a culture emanating from Peru, invigorated by the more forceful culture of Mexico, thence northward, might have spread over the land now occupied by the United States and Canada. Without printing and with only the quipus and the

picture characters of Mexico, progress at best would have been slow. To their disadvantage, too, the nomadic character of the Indian would not have built up a culture that the Arabs, the Greeks and the Romans gave to the Old World."

## BIBLIOGRAPHY

1. Bradley, Will T.: Medical Practices of the New England Aborigines, *J. Am. Pharm. Assn.*, February, 1936, pp. 138-147.
2. *Badianus Manuscript, an Aztec Herbal 1552, Codex Barberini Latin—Vatican Library*, translated by Emily Walcott Emmart at the instigation of the Smithsonian Institution, published in 1940. Johns Hopkins Press, Baltimore.
3. Brooks, Harlow: Navajo Indian Medicine, *Am. J. Surg.*, 31: 184, January, 1936.
4. Corlett, William Thomas: *The Medicine Man of the American Indian and his Cultural Background*, C. C Thomas, Publisher, 1935.
5. Moodie, Roy L., *Annals of Medical History*.
6. Stone, Eric: Surgery among the North American Indians, *Am. J. Surg.*, 13: 579, 1931.
7. *Ibid.*, *Medicine Among American Indians*, P. B. Hoeber, Inc., 1932.
8. Youngken, Heber W.: Drugs of the North American Indians, *Am. J. Pharm.*, series of articles in 1924 and 1925.
9. *Med. J. & Rec.*, May 20, 1931, page 509, from a Canadian paper. Miscellaneous books and magazine articles on aspects of Indian life which make incidental allusions to aboriginal medical practice.

## TEACHING TECHNIQUES FOR SLOWER STUDENTS IN ARMY COLLEGE TRAINING PROGRAMS

G. D. McGRATH

*Boulder High School, Boulder, Colorado*

When many army college training detachments began their academic programs, a number of instructors were employed who had taught on the high school level within recent years. Inasmuch as most of the army students to be instructed were highly selected, enlisted men, such programs seemed to offer a veritable proving ground for methods of teaching. Doubtless every trick in the bag came up for trial, re-trial, and evaluation. From such a motley array, probably could be cited a few which have proved to be the most effective procedures.

In some of the programs where men were segregated into class sections, largely on the basis of psychological or mathematical examinations, there often would be a class very similar in limits of ability to the frequently labelled "dull-normal" classification. At the outset, army authorities gave very few specific requirements regarding methods or techniques. They

were interested in getting a job done quickly and efficiently and left means to the end largely up to the ingenuity of instructors. Their theory was to salvage every logical candidate possible, train him, and make of him an asset to Uncle Sam's armed forces. This was to be accomplished through provision for additional academic background. The academic background of trainees before army experiences varied from a seventh grade education to that of two years of college. However, the average or mean schooling of the first 1500 men in the program in question, who filled out questionnaires, indicated on month of college beyond a high school diploma.

It is true that many educators have frowned on the use of the expression "dull-normal," but it has often been used to typify groups of students in academic-type classes who are below average in abstract intelligence.

There may not be definite lines of demarcation which separate these groups, and it is probable that without such limitations, the use of this type of term may not carry the same connotation for all. In spite of this, there is a level of intelligence in homogeneous grouping which we often broadly refer to as "dull-normal" and to this end, many trained investigators have set about to establish lists of teaching aids, techniques, and suggestions for more effective instruction for them. Teachers have been using these lists or modifications of them with great success for a number of years.

A situation soon developed whereby many instructors recognized the similarity of some of their former high school dull-normal classes to certain of the lower ability army class sections. One usually thinks of an accelerated army training program as a place where special techniques would be inadvisable. At least, extensive experimentation on the part of an instructor appeared out of bounds. Rather, a common assumption seemed to be to thrust subject matter at the trainees and make them master it—one method sufficing for all.

The writer decided to investigate possibilities of improving the learning situations for some of these lower groups in the Army Air Force Program, and to apply many procedures and techniques which he had worked out and used to advantage in teaching dull-normal groups in previous experiences. Many of these same procedures may be effective with any type of group, but some of them, at least, would not be satisfactorily applicable to above-average groups.

After nearly a year's trial in physics classes, it became apparent that these techniques produced a far superior achievement than the methods involving routing regular procedures. A list of simple suggestions applied to college Army Air Force training groups follows:

1. Begin discussion of a topic with very simple, obvious and familiar information. This reduces fear and inspires confidence. Permit the student to bring in personal background wherever it applies. The so-called dull-normal student gives excellent reports, too, and this forms a good point of departure for introducing many new topics.
2. Keep driving straight to the objective of the topic. The tempo must be adjusted to their ability to follow. Extraneous material and digression of the instructor to sideline topics, if any, should be deleted until the topic has been developed. Lower groups cannot lose their focus by diversion and quickly regain their ability to follow.
3. Provide practice in estimation wherever possible. Lower groups are prone to indulge in excessive snap judgment, but need training in improving estimation—and they enjoy it. It is difficult for them to see the absurdity of an illogical statement.
4. Activities should be changed very often. Their span of attention is short. Often a change in procedure is advisable every seven to ten minutes, or at even shorter intervals.
5. A simple task or set of tasks should be required for them to perform outside of class for nearly every class meeting. It is important that this assignment be over material covered that day in class, not on advance topics. It should be within the general mental limits of the group.
6. The goal of a unit should be clear at the start and realized when achieved. Their understanding of a topic should seldom be taken for granted. Instead, a few sampling questions should be asked to ascertain their comprehension.
7. Material should be "boiled down" carefully. It is difficult to select how much they can do. But all should master a list of minimum essentials based on the objectives and program of study.
8. The student should be permitted to dramatize as often as possible with himself the chief actor. If he can visualize himself, for example, in a P-38 with a Zero on his tail, or something similar, the work becomes much more interesting and vitalized for him.
9. The instructor should use a variety of well-prepared types of questions. Few, if any, classroom practices are more tiring than questioning in the same form of statement. A question should be stated clearly, once, and seldom repeated. This stimulates ability to take directions. Answers should be made in a clear statement. Seldom should a student be permitted to get off with "I don't know." Rather, the instructor should come back at him from a different

angle until he has made a contribution to the class discussion. Provide for answering each question before leaving the topic. There are at least fifteen distinctly different types of questions which can be used effectively to make discussion more worthwhile and interesting.

10. The class discussion should be paralleled with a written outline on the board which they are required to copy into their notebooks. Frequently, to stimulate organization and utility, they should be required to refer to something in the outline which was covered at an earlier date.
11. Frequent summaries should be made by the students. Let each one explain or summarize that area in which he feels best able to make contributions to the class. They like to get the feeling of contributing something worthwhile to class progress.
12. A frequent barrage of short answer tests of many varieties should be used. They should be regarded more as teaching devices than as means of marking. Often the questions should be read orally to the group to stimulate the ability to take directions and think at the same time.
13. New words should be printed, marked, or sounded out on the board. Very frequently, the less apt students are poor readers and poor listeners; especially with new terminology.
14. As many visual aids as possible should be utilized, such as films, slides, sketches, labelled drawings, etc. Much of their learning is through the eyes, not through the ears.
15. It is never wise to permit buck passing. These students are keenly aware of their deficiencies and like to explain it on the basis of an ineffective course in high school. It should be explained that they are there to learn all they can and to profit together to the greatest extent. Most of the so-called previous deficiencies can be eliminated by a little extra explanation and practice.
16. Commend honest efforts, even if below par. The student is frequently on the defensive and this helps him remove such barriers.
17. They should understand that major principles are the important things in conquest of knowledge, not facts alone. Great numbers of simple applications should bolster each principle—often as many as 50 applications based on a principle can be profitably discussed. Many of these applications should include current developments. They are eager for a modern and functionally-applied treatment.
18. It is seldom wise to send many to the board for problem work. They seem to do better by working at seats and checking with an instructor's work at the board. An occasional well-placed computation error by the instructor does wonders in keeping them on their toes and alert.
19. It is wise to have a short personal record before the instructor at all times. This can be jotted on a  $3 \times 5$  card, using a separate one for each individual. Such information as past educational background,

fields of major interest, vocations, etc., are very valuable for an instructor. Also handy is a notation about how well the student is doing in other subjects. The students seem to appreciate use of these cards by the instructor for calling on them to recite, with frequent shuffling to get an unbiased sampling.

20. A great deal of busy seat work for drill purposes will usually not be very helpful. Drill sheets may be effective, if not overdone. The students desire to see a reason for everything and are quick to question fill-in materials. Drill alone will not bring the desired success.
21. Such "dull-normal" like to measure their progress. A test before and after a unit to show progress is especially helpful. They like to keep score all along the way, whenever they are visibly evaluated by a score. They experience difficulty in evaluating their own efforts, unless some form of score keeping is encouraged.
22. The students should seldom feel any semblance of penalties or losses due to lack of intelligence. It is unwise to provide any situation which may cause them to feel accused of laziness. Many of them like to pretend that they are lazy, but resent any notice of it by instructors.
23. Such students appreciate more than others a genuine and sympathetic interest in their welfares. They react very favorably to human interest stories and humor, where applicable. However, they find it difficult to take a joke on themselves with grace and decorum.
24. Even though slow to perceive, they like action and like to keep things moving. Any methods which arouse curiosity pay huge dividends.
25. Very careful planning is needed more with dull-normal groups than with others. Their deductive powers are slower. Transfer of training cannot be relied upon. They have poorer memories and less associative recall. The best attack is characterized by its simplicity and definite planning for every minute of the class time.
26. They should be encouraged to teach each other when study conditions permit. This provides mutual benefit for learner and helper.

Some of the techniques found worthwhile on the high school level were impossible to apply to army groups. This is partially due to the nature of some army academic programs. Among these are:

1. Give positions of honor to challenge them. The army has its own system which obviates this.
2. Encourage hobbies. They have no time to think about this unless it involves post-war suggestions.
3. Encourage more extra-curricular work. Very few want any additional burdens, but it might work under somewhat limited circumstances.

4. Use more supplementary texts and materials. They will do well to master the adopted text, although an instructor might well refer to some material in other texts, especially for the more able students.
5. Let physical activity supplant much learning where feasible. They are too willing to let mere physical activity substitute entirely for learning.
6. Encourage recreational reading. They have too little time. Mere mention of it often depresses them.
7. Oral reading in class aids comprehension. While they may be poor readers, the time can be better spent in discussion and application rather than in oral reading.
8. Use much time for teacher demonstrations. They enjoy demonstrations immensely, but it is too time-consuming in accelerated programs.

We must remember that we are still working with a human individual who is not very much different after all than he was as a high school pupil in civilian life, and we can make and mould him perceptibly to his advantage.

This improved learning may well be the difference between life and death for many trainees. When it comes to pitching themselves out of a hole, and the answer book is not handy, many of our army students will find use for all the extra knowledge they may have acquired, And it is up to instructors to do whatever they can to insure this extra knowledge and the ability to use it functionally. The real important issue is that these lower groups can learn and they like to learn when given the proper situation. Tragically enough, somewhere along the way, too many of them have been given the impression that they cannot learn. Once they realize they can learn, and that learning is fun, they will lose much of their acerbity for education, and will become better citizens of this great America.

Thus it seems plausible that many of these methods and suggestions might be helpful to high school and college instructors in dealing with similar-type groups. We, as educators, have real challenges to meet in re-examining our premises and methods to the end that more of our citizenry shall profit through better educative experiences. When all educators realize that special methods pay such huge dividends, perhaps we shall not be guilty of as much vacuous teaching as some caustic critics imply, and we shall be headed toward a new era in educational achievement.

---

# THEORY OF ITERATED TRIGONOMETRIC FUNCTIONS

## PART I. THE ITERATED SINE

W. RAYMOND GRIFFIN

*120 Church Street, Ferguson, Missouri*

### INTRODUCTION

Literature on this function is practically nonexistent. The only known reference<sup>1</sup> is a doctor's dissertation in the original German. The only known copy of it, in this country, is in the Princeton University Library.

The subject warrants more attention and development than has hitherto been accorded it. It is possible that the theory could be so developed as to provide convenient solutions for types of differential equations, other than those already standardized, occurring in the theory of vibrations and known as "non-linear mechanics."

The following is an attempt to develop a theory of the iterated sine from the viewpoint of applied mathematics, and to deduce some of the geometrical and physical interpretations and implications; also, to present some empirical forms of the iterated sine which, it is hoped, will bring this function more into the ken of the applied mathematician and research engineer.

Nothing has been used from M. Koppe save the mere presentation of his equation for the iterated sine and several numerical results, and the equation is given in the appendix.

### DEFINITION

The iterated sine will be defined as follows:

$$\begin{aligned} f_1(\theta) &= \sin \theta \equiv \sin_1 \theta \\ f_2(\theta) &= \sin (\sin \theta) \equiv \sin_2 \theta \\ f_3(\theta) &= \sin \{ \sin (\sin \theta) \} \equiv \sin_3 \theta \\ &\dots \dots \dots \\ f_n(\theta) &= \sin \sin \dots \sin \theta \equiv \sin_n \theta. \end{aligned}$$

It will be convenient to think of the iterated sine as an operator. Thus,  $f_n(\theta) = \sin_n \theta$  represents the sine operating  $n$  times on the independent variable  $\theta$ . From this point of view,  $\sin \theta$  is a special case of a more general type of function.

## GEOMETRIC INTERPRETATION

The iterated sine may be geometrically interpreted as follows.

Consider a circle of unit radius as shown in Fig. 1. Suppose the radius makes a fixed angle  $\theta = \theta_1$  with respect to the abscissa  $OA$ . Then the ordinate  $\overline{BC} = 1 \sin \theta = \sin \theta$  from the definition of the sine of an angle. Now suppose the line  $\overline{BC}$  be picked up and so placed that the point  $C$  coincides with the point  $A$  and, further, that  $\overline{AB}$  lies along the circle. The point  $B$  will now lie at some point  $E$  on the circle. The arc  $\widehat{AE}$  now defines another

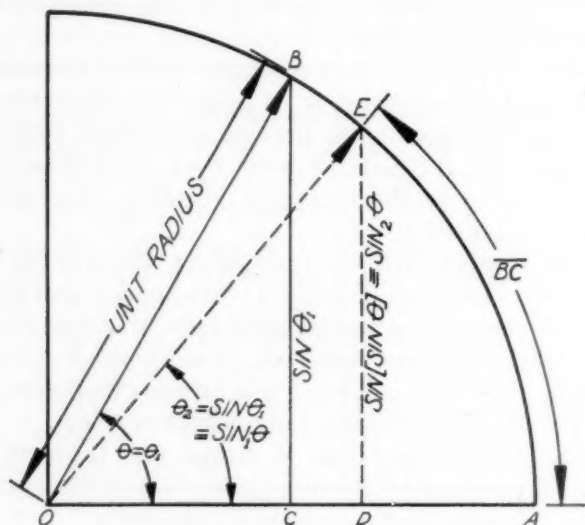


FIG. 1

angle  $\theta_2$  which angle is numerically equal to (in radian measure) the arc  $\widehat{AE}$ . This follows from the elementary relationship  $S = r\theta$  where  $S$  is the arc,  $r$  the radius, and  $\theta$  the central angle in radian measure. Here  $r = 1$ , and so  $S = \theta$ . But  $\widehat{AE} = \sin \theta_1$ . Therefore  $\theta_2 = \sin \theta_1$ , and  $\sin \theta_2 = \sin (\sin \theta_1)$ . This last defines a new ordinate  $\overline{DE}$ , etc., for as many times as one pleases.

(Note: All angles above and hereafter will be understood as being in radian measure unless otherwise explicitly stated.)

NUMERICAL EVALUATION OF  $\sin_n \theta$ 

The obvious way of doing this is, of course, to look up the sine of the desired angle. This sine is considered a new angle in radian measure. If radian tables are used, the sine of this radian

value is looked up, etc. This process is carried out the desired number ( $n$ ) of times, thus evaluating  $\sin_n \theta$ . If only degree tables are available, or if a slide rule is used that is marked off in degrees, then the additional step of converting back into degrees must be done, except, of course, the final value.

TABLE I. VALUES OF  $\sin_n \theta$ 

(The values below have been calculated using a 25 inch slide rule and are believed to be accurate to 3 units in the 4th decimal place. The values have been checked and rechecked. Only every 5th value above  $n = 10$  has been recorded.)

$n$	$\pi/2$	$\pi/3$	$\pi/4$	$\pi/6$	$\pi/12$
1	1.0000	.8660	.7071	.5000	.2588
2	.8415	.7617	.6495	.4795	.2559
3	.7455	.6895	.6050	.4615	.2530
4	.6781	.6360	.5690	.4450	.2502
5	.6275	.5940	.5375	.4305	.2475
6	.5868	.5600	.5120	.4175	.2447
7	.5535	.5315	.4900	.4055	.2420
8	.5255	.5070	.4710	.3945	.2395
9	.5015	.4855	.4540	.3840	.2372
10	.4805	.4663	.4385	.3747	.2351
15	.4060	.3977	.3800	.3360	.2240
20	.3585	.3525	.3405	.3080	.2146
25	.3250	.3205	.3115	.2870	.2068
30	.3000	.2960	.2890	.2698	.1988
35	.2805	.2775	.2713	.2548	.1930
40	.2642	.2610	.2560	.2421	.1882
45	.2498	.2478	.2427	.2320	.1832
50	.2385	.2360	.2320	.2222	.1795

Table 1 was thus evaluated on a 25 inch slide rule and the results have been plotted as shown in Figs. 2 and 3.

Another way of evaluating  $\sin_n \theta$  is a stepwise one. In the topic of Bessel Functions in Wood's *Advanced Calculus*<sub>2</sub> is found the expression

$$\sin(x \sin \theta) = 2 \sum_{k=1}^{\infty} J_k(x) \sin k\theta \quad (k=1, 3, 5, \dots). \quad (1)$$

For  $x=1$ , (1) becomes

$$\sin(\sin \theta) \equiv \sin_2 \theta = 2 \sum_{k=1}^{\infty} J_k(1) \sin k\theta \quad (k=1, 3, 5, \dots) \quad (2)$$

$\sin \theta$  may now be considered as the original  $\theta$ , and

$$\sin_3 \theta = 2 \sum_{k=1}^{\infty} J_k(1) \sin(k \sin \theta) \quad (k=1, 3, 5, \dots)$$

or, in general

$$\sin_n \theta = 2 \sum_{k=1}^{\infty} J_k(1) \sin(k \sin_{(n-1)} \theta) \quad (k=1, 3, 5, \dots). \quad (3)$$

For convenience a few values of  $J_n(1)$  are listed below.

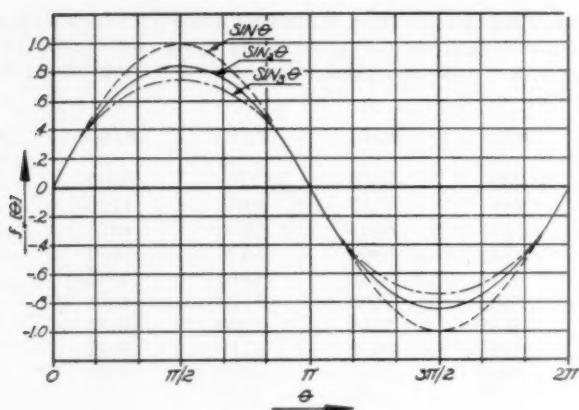


FIG. 2

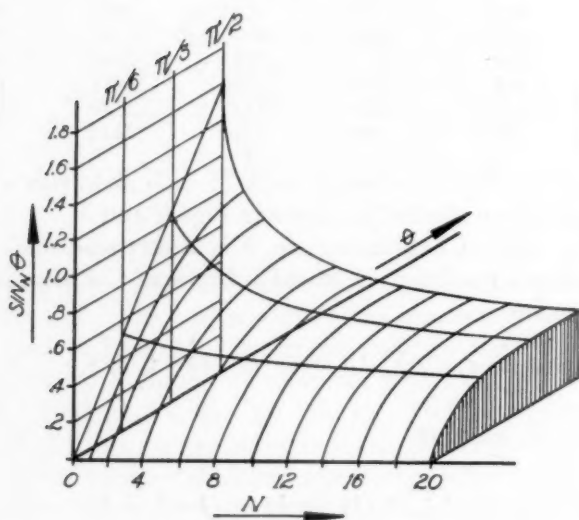


FIG. 3

TABLE II

$n$	1	3	5	7
$J_n(1)$	.4401	.01956	$2.498 \times 10^{-4}$	$1.502 \times 10^{-6}$

The above could have also been carried out in a more elementary form and without using Bessel functions. However, it is believed that this alternative form does not converge as rapidly. The alternative form is as follows: From the ordinary series expansion for the sine it is recalled that

$$\sin \theta = \sum_{k=0}^{\infty} (-1)^k \frac{\theta^{(2k+1)}}{(2k+1)!} \quad (k=0, 1, 2, 3, \dots).$$

In a similar manner as before

$$\sin (\sin \theta) \equiv \sin_2 \theta = \sum_{k=0}^{\infty} (-1)^k \frac{(\sin \theta)^{(2k+1)}}{(2k+1)!} \quad (k=0, 1, 2, 3, \dots)$$

and, in general

$$\sin_n \theta = \sum_{k=0}^{\infty} (-1)^k \frac{(\sin_{n-1} \theta)^{2k+1}}{(2k+1)!} \quad (k=0, 1, 2, 3, \dots).$$

#### COMBINATIONAL PROPERTIES OF THE ITERATIVE PROCESS

From the definition the iterated sine,

$$\sin_2 \theta \equiv \sin (\sin \theta) \equiv \sin_1 (\sin_1 \theta) \equiv \sin_{1+1} \theta,$$

similarly,

$$\begin{aligned} \sin_3 \theta &\equiv \sin_1 (\sin_2 \theta) \equiv \sin_2 (\sin_1 \theta) \equiv \sin_1 \{ \sin_1 (\sin_1 \theta) \} \\ &\equiv \sin_{1+2} \theta \equiv \sin_{2+1} \theta \equiv \sin_{1+1+1} \theta. \end{aligned}$$

Thus, by induction, it is seen that the commutative law of combination holds; i.e., it makes no difference as to the order of the iterative process.

It is further seen that

$$\sin_m (\sin_n \theta) = \sin_{m+n} \theta = \sin_{n+m} \theta. \quad (4)$$

If  $m = -n$ , then  $m+n=0$ , and

$$\sin_0 \theta = \theta. \quad (5)$$

The meaning of the zero subscript is that of taking the arc sine as often as the sine has been taken. Thus, the two operations are neutralized and we are left with the original quantity,  $\theta$ , we were to operate upon.

It is seen that the subscript notation is one that makes more consistent, logical, and compact the notation of writing angles,

sines, and arc sines, thus

$$\sin_{-1} \theta \equiv \sin^{-1} \theta$$

$$\sin_0 \theta \equiv \theta$$

$$\sin_1 \theta \equiv \sin \theta.$$

#### DIFFERENTIATION, WITH RESPECT TO $\theta$ , OF $\sin_n \theta$

This follows easily from the elementary properties of the differentiation of a function. The process is legitimate for  $\sin_n \theta$  is a continuous function of  $\theta$ . This last follows for it is known that the sine of a continuous, finite function is itself continuous. So

$$\frac{d}{d\theta} \sin \theta = \cos \theta$$

$$\frac{d}{d\theta} \sin_2 \theta \equiv \frac{d}{d\theta} \sin (\sin \theta) = \cos (\sin \theta) \cdot \cos \theta$$

.....

$$\frac{d}{d\theta} \sin_n \theta = \cos (\sin_{n-1} \theta) \cos (\sin_{n-2} \theta) \cdots \cos (\sin \theta) \cos \theta. \quad (6)$$

#### INTEGRATION, WITH RESPECT TO $\theta$ , OF $\sin_n \theta$

The writer knows of no general solution of the above. For the definite integral

$$\int_0^\pi \sin_2 \theta d\theta$$

the solution is a special case of the Weber-Lommel<sup>3</sup> function  $W(n, x)$ , where

$$W(n, x) = \frac{1}{\pi} \int_0^\pi \sin (x \sin \theta - n\theta). \quad (7)$$

For  $n=0$ , and  $x=1$ ,  $W(n, x)$  reduces to

$$W(0, 1) = \frac{1}{\pi} \int_0^\pi \sin (\sin \theta) d\theta \equiv \frac{1}{\pi} \int_0^\pi \sin_2 \theta d\theta, \quad \text{or}$$

$$\int_0^\pi \sin_2 \theta d\theta = \pi W(0, 1) = 3.1416(.5687) = 1.78662 \quad (8)$$

where  $W(O, 1)$  has been taken from Ref. 3. The value of (8) is reasonable when compared with the known relationship

$$\int_0^{\pi} \sin \theta d\theta = 2.000 \quad (9)$$

because it is known that  $|\sin_2 \theta| < |\sin \theta|$ . Since the ordinate is less and the interval of integration is the same, then, of necessity, the area under the curve for this interval must be less. Hence, the value of (8) must be less than the value of (9).

#### A DIFFERENTIAL EQUATION SATISFIED BY $\sin_n \theta$ . ( $n=2$ )

$u = \sin_2 v$  is a particular solution of

$$\frac{d^2 u}{dv^2} + (\tan v) \frac{du}{dv} + (\cos^2 v) u = 0 \quad (10)$$

$x = \sin_2 \omega t$  is a particular solution of

$$\ddot{x} + \omega^2 [\sin \omega t (1 - x^2)^{1/2} + \cos^2 \omega t] x = 0. \quad (11)$$

#### EMPIRICAL FORMULAS FOR $\sin_n \theta$ . ( $0 \leq \theta \leq \pi/2$ )

The following formulas give results with a maximum error of 5% for the ranges in which they are defined. Most of the values in the defined range have an error of only about 1 to 3%. Despite the error, it is nevertheless deemed more desirable than the infinite series form given in Ref. 1 and repeated in the appendix. From the viewpoint of applied mathematics, there are two good reasons to prefer the empirical form. First, by inspection, one can see the trend of the function as  $n$  and  $\theta$  are varied. Second, it is not so tedious to evaluate the empirical form.

(a) For  $n \geq 1, \pi/4 \leq \theta \leq \pi/2$ .

$$\sin_n \theta = \frac{\theta}{\pi/2} \operatorname{ctn}^{-1} [n^{K^u} \operatorname{ctn} u] \quad (12)$$

where  $K = 3/2\pi$  was valuated from the known value of  $\sin_2 \pi/2$  as given in Table I, and  $u = (\pi/2/\theta) \sin \theta$ .

(b) For  $n \geq 3, \theta \leq \pi/6$

$$\sin_n \theta = \theta \left[ 1 - \frac{\left( \frac{\theta}{\pi/2} \right) n^K}{C + \left( \frac{\theta}{\pi/2} \right) n^K} \right] \quad (13)$$

where  $K = 3/2\pi$  and  $C = (\pi/4)/(1 - \pi/4)$  was evaluated from the known value of  $\sin_{10} \pi/6$  as read from Table I.

(c) We shall now generalize our iteration concept such that we have continuous iteration. Thus,  $\sin_n \theta$  will now define a function continuous in  $n$  and  $\theta$ , or  $F(n, \theta) = \sin_n \theta$ . Thus, we can now have such expressions as  $\sin_{1/2} \theta$ . So,

For  $n \leq 1$ ,  $0 \leq \theta \leq \pi/2$ ,

$$\sin_n \theta = \frac{\theta}{\pi/2} \operatorname{ctn}^{-1} \left[ n^{K[(\pi/2)/\theta]} \operatorname{ctn} \left( \frac{\pi/2}{\theta} \sin \theta \right) \right]. \quad (14)$$

M. Koppe gives<sup>1</sup>

$$\sin_{1/2} (1) = .9087088. \quad (15)$$

Substituting  $n = \frac{1}{2}$  and  $\theta = 1$  in (14) gives, using a 25 inch slide rule, the value

$$\sin_{1/2} (1) = .905$$

an error of only 0.33%.

The trend of the function  $\sin_n \theta$  can best be seen by a study of Fig. 3 which figure was plotted from Table I.

It is seen from (12), (13), and (14) that

$$\text{Case I} \quad \lim_{n \rightarrow 0} \sin_n \theta = \theta \quad (0 \leq \theta \leq \pi/2) \quad [\text{except as in (13)}]$$

$$\text{Case II} \quad \lim_{n \rightarrow 0} \sin_n \theta = 0 \quad (n > 0)$$

$$\text{Case III} \quad \lim_{n \rightarrow \infty} \sin_n \theta = 0 \quad \text{asymptotically} \quad (0 \leq \theta \leq \pi/2).$$

It is seen that these trends are the same as those of Fig. 3.; i.e., the empirical forms coincide with the true values for the limiting cases. (The true value of case III cannot, of course, be read from Fig. 3.)

#### DIFFERENTIATION OF $\sin_n \theta$ WITH RESPECT TO $n$

For  $\theta = \pi/2$ , (12) and (14) reduce to

$$\sin_n \pi/2 = \operatorname{ctn}^{-1} [n^K \operatorname{ctn} 1.000]. \quad (16)$$

This special case has been chosen due to the complexity of (12) and (14).

Then

$$\frac{d}{dn} (\sin_n \pi/2) = -\frac{K(\operatorname{ctn} 1.000)n^{K-1}}{1+(n^K \operatorname{ctn} 1.000)^2}. \quad (17)$$

Now

$$K = \frac{3}{2\pi} \approx 1/2, \quad \text{and} \quad (K-1) \approx -1/2.$$

Thus,

$$\frac{d}{dn} (\sin_n \pi/2) \approx -\frac{1/2(\operatorname{ctn} 1.000)}{\sqrt{n}[1+n \operatorname{ctn}^2 1.000]} \quad (18)$$

where the negative sign means the slope is decreasing as  $n$  is increasing. For  $n=0$ , (18) becomes infinite, and the drawing of the line  $\theta=\pi/2$  in Fig. 3, such that it is tangential to the ordinate, is justified.

For  $n=\infty$ , (18) shows the slope to be zero and approaching it asymptotically. It is easily seen from (12) that, for any other angle  $\theta < \pi/2$ , (18) becomes

$$\frac{d}{dn} (\sin_n \theta) \approx -\frac{1/2 \operatorname{ctn} u}{\sqrt{n}[1+n \operatorname{ctn}^2 u]} \quad (19)$$

where

$$u = \frac{\pi/2}{\theta} \sin \theta.$$

For  $\theta \rightarrow 0$ ,  $u \rightarrow \pi/2$  and (19) approaches zero. This is also borne out in Fig. 3.

#### THE FORM $\sin_n \theta / \sin_n \pi/2$ .

This may prove to be the most practical form of the iterated sine as

$$\left| \frac{\sin_n \theta}{\sin_n \pi/2} \right|_{\max} = 1.$$

For any  $n$ , the absolute value would then range between zero and unity as in the ordinary sine function. The only effect of  $n$ , then, is to change the shape of the curve. This is clearly shown in Fig. 4. It is to be observed that increase of  $n$  roughly corresponds to increase of the modulus " $k$ " in the elliptic function  $sn(u)$  as can be seen in Fig. 4.1, page 126, of Ref. 4.

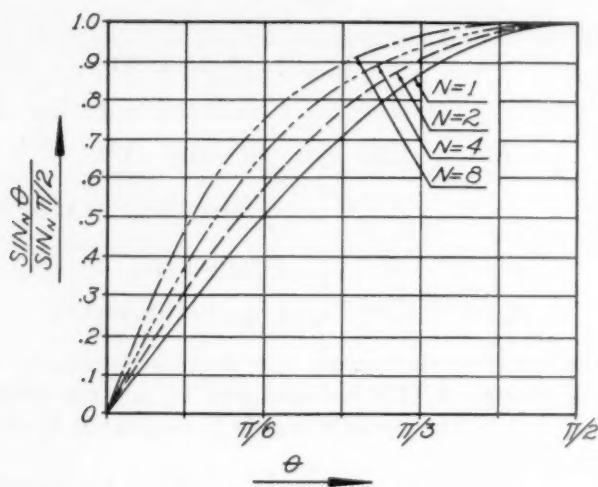


FIG. 4

## ACKNOWLEDGMENT

Generous thanks are due H. Clay Johnson for the preparation of the figures and to Harriet Griffin for typing of the article.

## APPENDIX

For the convenience of those sufficiently interested, the equation of the iterated sine as given by M. Koppe is given herewith:

$$\sin_n \theta = \theta + B_2 \theta^2 + B_4 \theta^4 + B_7 \theta^7 \dots$$

where the  $B$ 's are functions of  $n$ . The first three are

$$B_2 = -(1/6)n$$

$$B_4 = -(1/30)n + (1/24)n^2$$

$$B_7 = -\frac{41}{2^2 \cdot 3^3 \cdot 5 \cdot 7} n + \frac{1}{3^2 \cdot 5} n^2 - \frac{5}{2^4 \cdot 3^3} n^3.$$

The original article lists the  $B$ 's up to and including  $B_{15}$ . Koppe points out that the coefficients of  $n$ , and its powers, in the  $B$ 's are combinations of Bernoulli's numbers.

## REFERENCES

1. Koppe, M., *Die Iteration des Sinus und Anderer Funktionen*, Berlin (1909).
2. Woods, F. S., *Advanced Calculus* new edition, page 281. Ginn & Co.
3. Jahnke and Emde, *Tables of Functions*, page 211, Dover Press (1943).
4. Karman and Biot, *Mathematical Methods in Engineering*, page 126, fig. 4.1. McGraw-Hill Book Co., Inc., New York (1940).

## OUR WARTY ASSISTANTS

PERCY A. MORRIS\*

*Peabody Museum of Natural History, Yale University, New Haven, Conn.*

My earliest experience concerning a toad occurred when I was about ten years old. I had an uncle who was outrageously proud of his rather large flower and vegetable garden. He used to pay us boys five cents for each half-bushel of weeds (packed down hard!) that we pulled during the summer months, and various other stated sums for odd jobs that helped to keep his garden neat and productive.

One day I discovered a toad sitting solemnly under the spreading leaves of a cauliflower, and, sensing a new source of income, I promptly killed it and carried it proudly up to the house, confidently expecting to get an offer of at least five cents a dozen for more of the same, but instead I got a good old New England "dressing-down," and my first lesson in natural history.

"Toads," my uncle declared, "are just about the best helpers us farmers got!" He had read in some government publication or almanac that a toad will eat a certain number of cutworms in a night, and, figuring on the number of tomato, cabbage, pepper, and other plants that a single cutworm might destroy during its span of life, and their ultimate value if left to fruition, he had arrived at the amazing conclusion that each toad in his garden saved him \$11.85 per year, from this pest alone. No, he was not interested in dead toads, but he would, and did, pay me a penny each for live ones, up to two dozen, which he decided was about the proper number for the area of his garden!

To most of us, I am afraid that the toad is merely an ugly, uninteresting creature. We can't eat him, his hide has no commercial value, and he is worthless for bass bait. In a vague sort of way we know that he is doing no particular harm as he hops contentedly about his business. He doesn't eat fruit, and destroys no growing crops, so we tolerate the toad, and, excepting when an individual falls into our well or manages to get himself mixed up among the whirling blades of our lawn mower, we seldom give him a second thought.

We very properly give a great deal of credit to our song birds for their efforts towards keeping down the bug population, but

\* Author of *They Hop And Crawl*, recently reviewed in this JOURNAL, published by The Jaques Cattell Press, Queen St. and McGovern Ave., Lancaster, Pa.



American toad, *Bufo americanus*, calling from *They Hop and Crawl* courtesy The Jaques Cattell Press

too many of us are prone to overlook the fact that our toads are doing a comparable work, and in many cases doing a far better job. Most of the toad's hunting is done after sundown when the birds have knocked off for the day, and a whole host of insects is getting ready to get in their best licks at our prized plants.

Competent herpetologists have estimated that during the three months of spring and early summer planting a toad will consume about 10,000 insects, and that of this number close to 2,000 will be cutworms. If the damage done by a single cutworm be put at an extremely low figure, it will be evident that my uncle's estimate of the toad's worth was very conservative.

Cutworms are not the only injurious insects on the toad's menu, however. The larvae of tent caterpillars, gypsy-moths, and army worms, as well as adults of practically all of our crop-pests, are gratefully accepted by our warty friend of the garden. To a toad, anything that moves and is not too big to swallow, is entirely suitable for food.

It does not even draw the line at slugs, those slimy mollusks of the lettuce-patch that are studiously avoided by our better birds. The story is well known of the lady who attempted to produce a garden on a small toadless island off the coast of Massachusetts. Despite hours of back-breaking work in hand-picking the slugs from the leaves of her growing things she was staging a hopeless fight, until someone suggested importing a few toads from the mainland to help out. Sixty specimens were liberated on the island, and as the summer went on they "grew fatter and fatter" and her garden became a thing of beauty.

It is amusing to watch a toad out hunting. Let us say he comes across a rotting pear on which the flies and ants are holding a convention. Our toad stops hopping a short distance away and slowly crawls up to within a few inches. He opens and shuts his mouth and a fly or ant disappears. He repeats the operation and another insect is gone. It takes very close observation and unusual eyesight to see what is taking place.

The toad's tongue is not fastened at the back but is attached by its tip to the front part of the lower jaw. When he opens his mouth the tongue is hurled out like the principal character in a Jack-in-the-Box, and the unfortunate fly is held a prisoner by the sticky secretion with which it is covered. The tongue pops back instantly, carrying the fly to its proper destination, and our toad looks calmly about for the next victim. One is a little as-



Towlu's toad, *Bufo fowleri*, calling from *They Hop and Crawl* courtesy The Jaques Cattell Press

tonished to find such lightning-like movement associated with the slow, lethargic toad.

During most of the year toads live solitary lives, caring absolutely nothing for, or resenting the presence of, other toads in their territory. In April or early May, however, they gather in large numbers in the nearby ponds and marshes. Here they paddle about for a few days, adding their musical trill to the other welcome sounds of spring. Only the males do the singing, and during the trill their throats swell to alarming proportions.

The eggs differ from those of frogs by being in strings rather than in masses. Tiny "pollywogs" replace the eggs in a few days, and by mid-July the borders of ponds are alive with toadlets, each about the size of a rugged honey bee.

Toads are capable of expelling a milky fluid, which is secreted by glands in the skin. This secretion is harmless to the skin, but apparently it is very disagreeable to the taste; at least a dog will seldom pick up a second toad. Nevertheless, the toad has its share of natural enemies. Owls, hawks, and crows, as well as domestic fowls, feed upon them regularly. Skunks are one of the few mammals they have to fear, and they have been observed to rub the amphibian vigorously on the ground before eating it, apparently to get rid of most of the distasteful fluid. By far the worst enemy the toad has, however, is the snake. Some snakes, such as the flat-headed adder (hog-nosed snake), feed almost exclusively upon toads.

Needless to say, you may handle toads without any fear of contracting warts. The "warts" of the toad's back bear no relation to the warts to which the human skin is subject. The myth of the warts belongs in the same category as the tale of the hoop snake.

The economic value of toads is now well established, and is recognized by every experienced gardner. To those Victory gardeners and others with little experience, and with scant knowledge of the "balance of nature," I would say consider the warty assistant in your back yard. Treat him well, and encourage his presence. He will repay you with destroying more injurious insects than a pair of birds, and, remember, the toad will not steal your cherries!

---

With the current restrictions on vehicle and tires, accidents involving defective vehicles increased from 8% of the total in 1941, to 10% in 1942, and to 15% in 1943, according to the National Safety Council.

## A HISTORIAN VIEWS SCIENCE

LOUIS MARTIN SEARS

*Purdue University, Lafayette, Indiana*

Men of science, I greet you. It is an honor to participate in your deliberations. You are met to further the historic record of scientific gains in your respective fields of study. Your action bears witness to the ripeness of those studies. For the scientist, like any other man of action, like any creative artist, is more interested in fresh achievement than in past accomplishment. He seeks new worlds to conquer. Each major discovery opens ever widening vistas of research. The vineyard is large and the workers are few. How justify a record of the past when every man is needed for the blazing of fresh trails? It is only when cumulative results are large that it becomes imperative to find some means for their recording. The sheer weight of past accomplishment summons the scientist to the labor of assessment. Committed to this enterprise, he finds that other disciplines are faced with similar necessities. Medicine and pharmacy, chemistry and botany, physics and geology, to mention but a few, are not the only fields of inquiry to attain an era of self-consciousness. But science as a whole acquires a new perception of its impressive contribution to the record of our race. Your presence here to-day is proof that you whose interests are so varied represent curricula so rich and fruitful that to deny them their just place in history would be unfair to your coworkers of the past and present.

It is here that the historian by profession may venture a suggestion, for his problems, long in contemplation and solved in part, at least, are not without their similarity to yours. What, then, can he suggest to the scientist turned historian temporarily? But first, it may be advisable to define the historian. Is his field of inquiry a parish or an empire? Is he immersed in antiquarian detail, or does he seek from scattered data to evolve a philosophy of history? Does he rejoice in each new fact? Is he in ecstasy over the discovery of a body of fresh records? Does he add, however unimportantly, to the body of existing knowledge? Or is he more concerned with the reinterpretation of

---

An address delivered at the opening meeting of the Indiana Academy of Science on November 10, 1944. Submitted for publication in *SCHOOL SCIENCE AND MATHEMATICS* upon the request of the General Biology editor. J.E.P.

facts already known? Does it please him to debunk a hero or to reinstate a victim of past disapproval? In either case a reversal, one can see, of verdicts previously reached! Desirous of success within his chosen field, will he measure that success by new facts freshly garnered, or, to use a Biblical analogy, will he pour the old wine into new bottles?

Reputable historians, it should be added, may be found in all these varied categories. It is no aspersion of an historian to define his interests as narrow. Not everyone can or should be an H. G. Wells, interpreting the whole range of man's past record. To prepare a single brick for an edifice so vast is not to be despised. The historian of a parish or a university, of a college fraternity or of a regiment, adds his due quota to the sum of knowledge. Awareness of the details thus assembled may contribute to the broader vision of a Gibbon or a Spengler. So with the scientist turned temporarily to history. No fact is too minute for the recording. But ever there will be those whose gift is synthesis and who can merge great bodies of detail into a sweeping generalization that may illumine a whole era with a blaze of hitherto undreamed perception.

Revolutionary changes of this sort occur in pure historical interpretation as influenced by scientific attitudes. American history, for example, was interpreted from earliest colonial days until well past the middle of the Nineteenth Century in terms of a scientific and political concept known as the theory of compact. Nor is that theory at all fantastic. Much evidence in fact supports it. Occasions can be noted where compact created not only the London and the Plymouth companies which founded Jamestown and Plymouth respectively, but where, as in the Mayflower Compact, signed and sealed before the Pilgrims landed on their rock, a government was actually founded in terms of compact. Once accept the theory of compact and a vast array of data falls into place. The political philosopher and the historian envisage man in the remote beginnings of society as in a state of nature, to employ their customary phrase, in other words a state of anarchy, each person his own ruler. Anarchy being obviously intolerable, government arose by voluntary compact. Much philosophical speculation, particularly in revolutionary England of the Seventeenth Century where one king lost his life and another lost his throne, argued whether compacts were or were not capable of termination. One school of thought, the revolutionary, held that a com-

pact, no matter how fully ratified, could be voided by any obvious breach by either party. A more conservative school of thought, led by the philosopher Thomas Hobbes, argued that no matter how regrettably the King might seem to breach the compact, his subjects were indissolubly bound by it. King and subjects were permanently one.

If compact thus maintained the structure of society in England, it underlay with equal force the empire as a whole. The system received, therefore, a mighty shock when the Americans broke their ties of allegiance to the mother country. But the theory received new impetus in the articles of Confederation and the federal Constitution. The signing of these documents represented quite definitely a compact. And within the framework of the government thus sponsored, debate was speedily renewed upon the immutability of contract—a debate not settled until the American Civil War, which on many a bloody battlefield determined that ours was an organic nation, or, as we to-day affirm, a "Republic one and indivisible."

It is not my purpose on an occasion like the present to survey any particular historic period, but rather to set forth a respected system of historical interpretation which was swept away completely by scientific change and revelation. It is probably far more than a coincidence, but if coincidence alone assuredly a most interesting coincidence, that the American Civil War of 1861 to 1865 followed by only two years the publication of Charles Darwin's *Origin of Species*. This work of pure science but of infinite imagination not only set in motion new currents in the warfare between science and religion but it insured the final overthrow of compact as the basic theory of human organization, substituting for the theory of compact an organic concept of society in which all the members were subordinated to the head. This head, it may be added, could by no remotest chance, resemble that of Cerberus, the three-headed dog that guarded access to the Lower World—and equally by no remotest chance could it resemble the Eighteenth Century concept of three equal powers, executive, legislative, and judicial, in the framework of the federal Constitution. In other words there must be one head by nature's ordinance, not three by man's contractual enactment.

Thus Darwin, the pure scientist, and others who have expanded and modernized his concepts, have wrought profoundest changes in the social, the religious, the political opinions of West-

ern civilization. His pioneering has been of infinite concern not only to the sciences in which he labored, but to the entire modern era, wherein Nineteenth Century liberalism is a waning force, and wherein the centralization that produces a Stalin, a Hitler, a Churchill, or a Roosevelt, seems to post-Darwinian man a natural phenomenon.

These remarks upon Charles Darwin have been concerned primarily with some changes in political theory which arose from his hypotheses. The consequences for theology were at least as startling. The Dante-Milton concept, derived from Bible sources, of the fall of man from an estate which angels might have envied to the lowest hells the poets could envisage, suddenly gave way to the rise of man from ooze and slime and protozoic limitations, to a status only slightly lower than the angels. It was revolutionary and bewildering and great souls were stirred profoundly.

Other scientific revolutions have been accompanied by comparable convulsions in theology and with them general changes in man's whole concept of existence. Of these, few have been so startling as that revolution in astronomy which stopped the sun from its rotation round the earth and set the earth instead to spinning round the sun. It is worthy of note that the astronomers whose names are forever identified with this truly cosmic change, Copernicus, 1473-1543, and Galileo, 1564-1642, lived respectively just before and just after the more dramatic phases of the Protestant Reformation. Now it would be unscientific to discount the economic, yes, even the theological implications of this religious revolution. But with due regard for these time-honored explanations, can one doubt that so complete a revolution as a flat earth suddenly becoming round, a God residing not much farther from his subjects than Zeus and the Olympians were distant from the Greeks, transformed with equal suddenness into a pantheistic deity pervading a universe measurable in terms of thousands of light years—can one doubt, I say, that an intellectual revolution so complete would stir profoundest repercussions, would even urge men to their death in their defense of ancient truth or revolutionary change? Certainly the Sixteenth Century revolution in astronomy cannot be divorced entirely from the Sixteenth Century revolution in religion. Thus science contributed to wide general change. Revolution could not be limited to a purely scientific concept. Passing into religious spheres, it stirred the entire emotional as well as

intellectual life of man—and its echoes still are audible.

Applications of science to military change are a constant theme in history. The possession of scientific information as yet unknown to the enemy has generally spelled victory in war. Greek fire in antiquity was the gift of chemistry to military strategy. The crossbow, the arquebus, the gun in its successive adaptations; the trireme, the galleon, the Royal Harry, the superdreadnaught, and finally the airplane carrier have given due advantage to their owners. But we are here in the field of invention, a corollary of pure science rather than its typical expression. In our immediate day the scientific laboratory has won the war in the Pacific by its success with rubber substitutes. Japan in 1941 might doubt the successful waging of a modern war with basic rubber sources unavailable. She is no longer comforted by any such illusion. Science is winning this war. The consequences are incalculable for all mankind. Certainly they will represent an impressive chapter in the history of science.

As the historian views the present war and its precursor of 1914 to 1918, it is essentially a conflict between slavery and freedom. It is the most serious challenge since the Sixteenth Century ushered in the age of revolution to the progressive forces represented in the Protestant Reformation, in the English revolutions of the Seventeenth Century, and in the American, the French, and latterly the Russian revolutions. For the modern age is the age of revolution. It is dynamic, in contrast with the static character of the centuries which intervened between the downfall of the Roman Empire and the voyage of Columbus. In these four and a half centuries of growing freedom for the human spirit, science has inevitably shared. When the obscurantist forces of counter-revolution finally gathered strength under William Second and Adolf Hitler to challenge the dignity of free nations and free men, science naturally lent its aid to the free world in which its own achievements had been so generously fostered. Which is not, of course, to say that the free peoples had a monopoly of science, or to imply that the forces of reaction had no scientific aids at their command. To the thoughtful the explanation goes quite deep. Certain forms of culture obviously survive even under tyrannical repressions in the general society. Great art, perhaps great music, is not entirely incompatible with social and institutional tyranny. Is that also true of science? Can it function at its best amid a

myriad "*verboten*"? Only the scientist can provide the answer. But the historian, viewing science as a major element in the general progress of society, inclines to answer No. When a government like Hitler's undertakes to say that those sciences which are concerned with racial evolution, genetics, for example, or ethnology, must preach the dogma of Aryanism according to the intuitions of a bigoted and ignorant dictator, not only are those particular sciences made to appear ridiculous, but the contagion spreads to the intellectual world as a whole and science bows its head in shame. Universities like Göttingen, dignified by centuries of high achievement, lose caste and standing in the general culture of mankind. And a whole nation falls into low esteem.

To the historian this statement of the case appears convincing. Fresh scientific data may seem to be forthcoming by a species of inertia from traditional sources, but their spring has been polluted. The ultimate consequence can hardly fail to be a general decline of scientific quality in harmony with a decline in civilization as a whole when gangsters and their goons presume to order the destinies of nations.

May there not be a hint in this for the general history of science? Your present speaker's familiarity with either history or science is not sufficient to provide the data needful for such a major thesis, but he would expect a properly qualified historian of science to find a conclusive coefficient between periods of human freedom and similar periods of major scientific progress.

Such a long range view of science as this correlation presupposes is suggestive of yet another major inquiry—namely the scientific relativity between the so-called "scientific" West and the contemplative East. Disregarding the Hitlerian concept of a "*Herrenvolk*" and recognizing only the essential equality of the races, it must be admitted that a great gulf exists between West and East with respect to their control of nature. Western man for centuries has devoted most of his energy to the conquest of his physical environment. Remarkable success has attended these efforts at subduing the material world. This cumulative accomplishment has been attributed in large measure to the introduction of the scientific method, what we to-day would call the laboratory method, by Roger Bacon, a Thirteenth Century friar who admitted as the truth only that which met the test of scientific inquiry.

Granting that one titanic intellect could impose upon his

fellows a method of this sort, an historian who subscribed as did Carlyle in "Heroes and Hero Worship" to the personal equation would reckon Bacon as a major subject for biography, for who in all these centuries has left a deeper mark than his? The student who rejects so personal an explanation may prefer some latent quality in Western man that made a Bacon certain to arise for the crystallizing of an instinct already planted in his fellows. Whatever the truth may be, Eastern man till very recently, and then apparently through contact with the West, has experienced no comparable urge. By our theory of race equality he has been endowed with talents equal to the Westerner, but he has employed those talents subjectively rather than objectively. Religious mysticism and philosophic speculation have been his substitutes for science. Great religions have been the major contribution of the east to the culture of mankind. It was only when the Westerner, having conquered his immediate environment, journeyed East in search of raw materials and eventually of empire, that Eastern man, brought forcibly into contact with his scientific brother, learned at first hand the glories of the scientific method, and the treasures latent in the test tube. That catalytic experience occurred some decades ago, and Japan and India, notably among Eastern countries, already have enriched the scientific world with valuable discoveries. They have not, it is true, snatched leadership in science from the more experienced West, but they have demonstrated that science is to them no longer a sealed book.

What lesson can the scientific West derive from these exchanges? The West has all along derived religious inspiration from the contemplative East. If the East henceforth is to glean objective mastery from the scientific West, the exchange of values may be better equalized. But what of that military ascendancy that science has bestowed upon the West? Can it be held in perpetuity against an emancipated East? A problem here is posed of most far-reaching implications. If further acquaintance with the scientific method should convert Eastern man from his subjective past to an objective future and should lead him to exploit the untapped resources of the Asiatic continent in an imperialism typically Western, the West would indeed be faced with problems of the utmost moment. Their best solution would appear to lie in a blending of Western materialism and Eastern mysticism in a joint possession of both hemispheres so happily

balanced that the spiritual element in man may eventually modify his more predatory instincts.

Having devoted some thought to rather sweeping generalizations suggestive of the broader approaches to the part which science plays in the unfolding record of the race, it may be permitted to turn our attention to some of the lighter *jeux d'esprit* in the scientific story, the antiquarian approach to science if one may call it such. For science has not always presented the systematic body of knowledge which it now connotes. Adult to-day, it was not ever so, and the childhood and the adolescence of science unfold some charming tales which the historian may narrate with perhaps a touch of nostalgia. For it would afford something approaching return to innocence to believe them—the innocence of Alice in her Wonderland, of Santa Claus and his admiring followers.

To put it in another way, the history of science like general history may be subdivided naturally into its ancient, mediaeval, and modern phases. The active scientist is necessarily a modernist, even a futurist. The historian of science, on the other hand, may find his most congenial field of research in Greek philosophy—for philosophy in antiquity was an omnium-gatherum for the learning of the day.

By no means least among the philosophers, that is to say scientists, of Ancient Hellas was Empedocles, a citizen of Agrigentum, a Greek outpost in Sicily, who lived from 490, or perhaps it was 483, to 430 B.C. Empedocles was a most distinguished person, of really towering intellect, who conceived of nature as an eternal reality but in its infinity of form assuredly not a unity. In his cosmogony, earth and water, air and fire, constituted the four elements which in ever shifting combinations exhibited perpetually the forces of attraction and repulsions, in other words of love and hate. Empedocles was primarily a botanist and zoologist, by some regarded as the founder of the Italian school of medicine. In his interpretation attraction, that is to say love, brought temporarily into existence such discordant elements as heads without necks, or arms without shoulders. Human bodies occasionally sported horned heads; while oxen for their part wore the heads of men. Sex, moreover, was frequently double. Innumerable oddities betrayed the fact that Nature was in experimental mood. It was a transient mood at that, for members soon found their proper partners, sex was

simplified, and organic life perpetuated, in what now appears to be a dim anticipation of the survival of the fittest.

If Empedocles may serve as our example of science in the Fifth Century B.C., we may glean some insight into science in the Middle Ages from the bestiary, which from the Second Century to the Thirteenth was so interesting to mediaeval man. The bestiary was a highly fanciful zoology, so real to its believers that it carries into modern folklore and to the pseudo-scientific terminology of our daily language. Mediaeval man knew many animals which we do not. It is not that they are extinct. They never did exist in the world of zoology. In the world of imagination, some of them still live. The bestiary linked with theology as nearly all things mediaeval did link. Hence, the peculiar religious symbolism of the unicorn. A well-known bestiary of the Thirteenth Century, published in book form at Caen, in Normandy, in 1852, remarks upon the single horn of the animal and adds that it alone of animals has courage to attack an elephant, which it does with the sharp nail of its foot rather than its dominant single feature, the horn. Still more remarkable is the life of the unicorn as a hunted rather than a hunting creature. In a forest which unicorns were known to haunt, a young virgin was set as a decoy. The unicorn, sensing the presence of the damsel, would seek her out and lie down at her feet, there to fall an easy prey to its pursuers. The moral then was drawn with characteristic piety: "The unicorn represents our Lord Jesus Christ, who, taking our humanity upon Him in the Virgin's womb, was betrayed by the wicked Jews, and delivered into the hands of Pilate. Its one horn signifies the Gospel truth, that Christ is one with the Father," etc.

The bestiary exerted a profound influence upon mediaeval art and heraldry and thinking. Shakespeare knew it well, and we to-day are not unmoved by the "crocodile's tears" as that saurian monster drowns a man all the while weeping unrestrainedly. More constructively, we approve the she bear's "licking into shape" of her amorphous young, born like the earth of Genesis, void and without form.

Indeed the zoological garden of a mediaeval bestiary which for many centuries instructed our ancestors in the mysteries of science can even now provide the most sophisticated modern with a nostalgic thrill of pure delight. While to the scientist turned historian it affords a rare field for research.

In the third periodic equivalent of general history, the mod-

ern, the sweep of scientific achievement has been so vast that even a hint respecting it would seem to the scientists here gathered a presumption on the speaker's part. Of no man since Leibnitz, 1646 to 1716, has it been said that he possessed all the knowledge of his day. Of Aristotle the same has been said with greater reason. It will never again be said of any man. For amid the vast stretches of knowledge extending in all directions, no one will ever again be master.

Thus it is likely that the historian of science, like the more general historian, will find two courses open to him. In the field of synthesis and wide generalization, a few examples of which were noted earlier in this paper, he may tie in the progress of science with world history in a manner which should be stimulating to each. Of if the more minute or antiquarian approach is the more available or affords the greater interest to the student, the historian of science will find, as the historian *per se* has long ago discovered, that history is like a mine. The field is so immense that the student can carve for himself a little niche, not unlike the miner's, in which he may find to his surprise, modest man that he is like to be, that he is actually the authority, not improbably the only authority now living, in the precise field of his research. Such a communion with the past will give him a more living realization of the laborious steps by which present knowledge has been reached, and a juster understanding of acceleration, wherein the portals of the future open in a splendid vision.

---

## RELATIONS OF TRIGONOMETRIC FUNCTIONS

LEONARD THATCHER

*Woodland High School, Woodland, California*

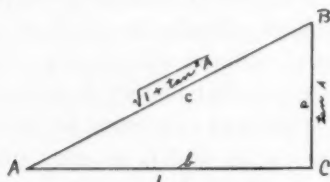
In proving trigonometric identities and in solving trigonometric equations, it is often necessary to write one trigonometric function in terms of some other function. Also in the integral calculus it is frequently necessary to make these transformations in order to get an expression involving trigonometric functions into a form that can be integrated or found in a table of integrals. In this school we feel that it is important that the student become proficient in making these transformations rapidly. We have developed the following short method by which one function can be written very rapidly in terms of any

other function. Even if the student should learn the thirty-six relationships, this method can be used as a check in case he is doubtful about any of them. We feel that this method might be helpful to other teachers.

Ordinarily, especially on the day that we teach the method, the following procedure is followed. Let us write the sine of an angle in terms of the tangent of the same angle. First we draw a right triangle and letter it in the customary way as shown below:

$$\tan A = \frac{a}{b}.$$

Take the denominator of the algebraic ratio which is equal to the function and give it the value 1.



i.e.  $b=1$ . Then  $a=\tan A$  and  $c=\pm\sqrt{1+\tan^2 A}$ , by the Pythagorean Theorem. Substituting the above values of  $a$  and  $c$  in

$$\sin A = \frac{a}{c} \quad \text{we have} \quad \sin A = \pm \frac{\tan A}{\sqrt{1+\tan^2 A}}.$$

As can readily be seen, the other five functions can also be written in terms of the tangent from the above figure; such as

$$\cos A = \frac{1}{\sqrt{1+\tan^2 A}}, \quad \csc A = \frac{\sqrt{1+\tan^2 A}}{\tan A}, \text{ etc.}$$

---

Based on mileage, special studies indicate that drivers under 20 years of age have the highest accident rate, the National Safety Council reports.

---

*During 1943, the equivalent of 221 average size classrooms of children between the ages 5 and 14 years were killed in accidents, the National Safety Council reports.*

## SCIENCE IN A NEW WORLD\*

ANDREW LEITCH

*Bethany College, Bethany, West Virginia*

There have been several extraordinary periods in the history of the world. There was the golden age of Greece which produced Socrates, Plato, and Aristotle. There was the rise of Christianity when a new notion of the value of the individual person and his relation to his fellows was born. There was the fall of the Roman Empire when chaos seemed to envelop civilization. There were the crusades with the growth of commerce and nationalism which followed in their wake. There was the liberation of the human spirit during the renaissance expressing itself in a rebirth of art, literature, religion, discovery, invention, science, and philosophy. There were the industrial and political revolutions of the eighteenth and nineteenth centuries with widespread economic, sociological and political repercussions. Finally there is our own incomparable age.

Ours is an age of mass production, popular education, social unrest and evolution, economic change and political upheaval. It is also an age of science. It is an era of science come of age in a world of change, in a world tottering precariously on the brink of chaos or at the foot of an upward climb unparalleled in human history.

Our age of science is not a new creation of our own. It is the culmination of a long series of contributions and advances from the time of Aristotle through Copernicus, Galileo, Kepler, Descartes, and Newton, through the long line of brilliant scientists of the nineteenth century including such illustrious names as Darwin and Pasteur, up to such wizards as Burbank, Carver, and Edison and a veritable galaxy of scientific stars of our own day.

The spectacular resurgence of science which we are now witnessing in the twentieth century has been stimulated profoundly by the two wars which have engulfed the world in our generation. Discoveries, inventions, advances of all sorts of a scientific nature which in the normal course of events would require fifty or a hundred years for their development have been compressed into the short span of four or five years. It was

---

\* Presidential Address, W. Va. Academy of Science, May 5, 1944.

so in 1914-1918. It is true again and will become increasingly so the longer the present war continues.

Without our scarcely realizing it we lived in a new world during the two and a half decades following 1918. Gone forever from the highways were the horse and buggy and the old fashioned dray, replaced by the passenger car and the motor truck. Trolleys and interurbans gradually gave way to buses, local and transcontinental. Even the railroads in both freight and passenger traffic were almost threatened with extinction until the present war over-taxed all transportation facilities and saved the railroads, for a time at least. With the automobile come of age other important developments followed. Improved highways spanned the continent. American families traveled as never before. The consolidated school was made possible. A mass migration of city dwellers began from the congested city centers to the suburbs and rural areas, with a consequent boom in the building trades. The use of the telephone was greatly expanded in both home and office. In the meantime whole new industries had sprung up almost over night, notably the radio and the motion picture, first the silent and then with sound effects. In the kitchen was installed the electric range and frigidaire. The automobile, the telephone, the radio, the frigidaire, the motion picture and silk hosiery became almost the symbol of American life in the two decades preceding Pearl Harbor. Needless to say they produced profound changes in the entire structure of our social life.

The present war will have repercussions far more significant than those of the last. The forces which emanated from that conflict and which gave us a radically different world in the twenties and thirties have been magnified a hundred fold. Inevitably we shall live in a new world in the post war era of reconstruction, one which defies the capacity of the human imagination to conceive in its totality. It is possible, however, on the basis of the present status of scientific achievement to foresee some of the more important developments which are bound to come.

In the realm of transportation the airplane will occupy a place comparable to that of the automobile in the past generation. The principles of mass production applied to the airplane for war purposes, along with the development of new designs of plane and engine utilizing high power fuels, plastics, and light metals which are now just in their infancy, together with the

perfecting of helicopter devices and parachutes capable of landing an entire plane in safety will place an airplane flivver in almost every back yard or roof-top. Our children will take to the air as eagerly as our generation did to the road. Only this time national boundaries will be obliterated. A business man will have supper in London and breakfast in New York. American families will week-end in Switzerland, and honeymooners will visit Alaska rather than Niagara Falls. Likewise, the flight of the Constellation recently on April 17 from San Francisco to Washington in 6/57/51, a ship weighing forty tons and capable of carrying 100 fully equipped paratroopers gives us a glimpse of what is just around the corner in air transport of both freight and passengers. "Airplanes now designed are capable of flying to Europe and back non-stop, carrying pay loads of 20 tons. . . . Designers are thinking in terms of hemisphere—spanning freighters and of passenger aircarriers in fleets numbering hundreds of planes. Transcontinental non-stop air trains of gliders . . . are no longer figments of an imaginative air man's dreams." (Quotation from Charles M. A. Stine: *Molders of a Better Destiny*, Science, Oct. 2, 1942, p. 310.) When as a consequence of this knitting of the world together socially and commercially the peoples of Europe and the Orient alike become our next door neighbors gone forever will be American or any other kind of isolationism, as surely as we witnessed a generation ago the passing of the horse and buggy. For this new world political science must devise new and more adequate forms of national and international government, economics must eliminate tariff walls, and sociology will be challenged to create a new social order in which brotherhood will prevail. The whole world will be closely knit into one big family, and as in all well ordered families, peace must be established on the basis of mutual understanding and good will. The recurrence of a devastating war once every generation must become as obsolete as small-pox. Either that or our civilization will perish.

The changes brought about by the development of the airplane and consequent new modes of transportation and communication are symbolic of other changes equally as revolutionary in other areas. By radio and colored television combined, the opera star will be brought realistically into our own sitting room. We will see every play of the ball-game as clearly as if we were in the stands. Vast new industries are being born out of war-time necessity—the aluminum and magnesium indus-

tires on a colossal scale, pliable and unbreakable glass, radar and electronics, high octane gasoline 50% more powerful than 100 octane aviation gasoline, fuel and oil made from farm products to run the world's automobiles after the present reserves in the ground have been exhausted. The houses in which we shall live will bear little semblance to those of today—pre-fabricated with sectional units and movable partitions, fire proofed, insulated, air-conditioned, electrified, heated by the sun, and with one way vision glass enabling us to look out but preventing neighbors and passers-by from looking in. Medicine also holds out the bright hope of the eventual conquest of disease including dreaded cancer and venereal diseases, through the application of the miracle sulpha drugs and penicillin. The new cure of syphilis in 24 hours, the 6 hour cure of gonorrhea, the immediate stopping of meningitis epidemics by sulphadiazine, the very recent prevention of seasickness in the armed forces, these are but portents of more miraculous things yet to come in the prevention and cure of disease, while continuous caudal anaesthesia bids fair at last to eliminate the age old pangs of child birth. These are but a few of the revolutionary changes that are our heritage for the world of tomorrow.

What function should science perform in this new world created to some considerable extent by science itself but created also in part by other forces—political, economic, sociological, educational, moral, and religious? In the first place, the scientist must continue as of old to search for the truth—open-minded, broad minded, tolerant, impartial, faithful to the facts. His motto must always be "Ye shall know the truth." His primary objective is first to discover and then to reveal knowledge to man. In order to accomplish this purpose, however, he must be free—free from fear, free from want, free to pursue his investigation, free to proclaim his findings to the world. To the four freedoms of the Atlantic Charter must be added, as far as the scientist is concerned, a fifth freedom—freedom of critical inquiry and research. For this reason science must help win this war in order to protect itself. It must fight to make the world safe for freedom of thought. Without this freedom the scientist cannot pursue his objectives. He must be free to explore any field, every field, to follow the facts and deduce conclusions based on the facts. His only coercion is that of the truth itself.

Totalitarianism is diametrically opposed to the scientific spirit. In this kind of political order scientists must support the

dogmas of the state. Instead of conclusions deduced from observed data, whether they support political doctrines or not, the scientist must subscribe to tenets that are politically expedient. Hence the preposterous doctrine of Nordic superiority and Jewish inferiority, and also the unseemly spectacle in Germany of some scientists conveniently accommodating the Nazi party by espousing its doctrines. To the glory of German science however, let us not be unmindful of the exodus of scientists from Germany away from the intolerable Nazi regime—Einstein, Stern, Koffka, Kohler, and scores of others. Our immediate task as scientists, therefore, is to help win this war.

In our world of tomorrow science will have a second function. In the past it tended to feel that it had fulfilled its obligations to itself and society when it had discovered knowledge and had shown its applications. In the new world of the post war era, however, science must help create a new social order. Scientists on the whole may no longer bury themselves in their laboratories and ignore social and political winds and currents. The Nazis have surely driven home this lesson. The scientist must be a citizen of the world. After he has discovered the truth he must not remain aloof and indifferent to the use of his own findings. He will be vitally interested in knowing whether they will be used for the weal or woe of mankind. He will realize that the ultimate objective of science is to solve human problems, to promote human welfare, that science at its best is cultural, designed and destined to enable man to climb the long and sometimes steep ascent to the richness and fullness of life.

The new physical world that science was in the process of creating before the Nazi holocaust science must defend and preserve. The new social world that will emerge when the smoke of battle has cleared away science must help to create. Science did not cause this war. The scientist is a man of peace, interested always in making this old world a better place in which to live, in promoting the highest well-being of man, in creating a social order in which justice prevails, with prosperity, happiness and brotherhood for all. These are the fruits of peace not war.

In the molding of this new social order which must arise out of the wreckage of our old world science must join hands with philosophy and religion. Along with the philosopher the scientist must co-operate in determining meanings, values and ultimate objectives. Along with the man of religion the scientist

must seek to promote the kindgom of God and of man in which is recognized the infinite value of personality, the brotherhood of all mankind, and the reign of love versus force. Then and then only will we have the kind of new world in which we want to live.

## NOTES FROM A MATHEMATICS CLASSROOM

JOSEPH A. NYBERG

*Hyde Park High School, Chicago*

**95. The Simplest Form of an Expression.** The simplest form of any expression depends on what we expect to do with that expression. If I want the square root of 80 and have a table of square roots at hand, then  $\sqrt{80}$  is a more desirable form than  $4\sqrt{5}$ . However, if my table includes only the numbers from 1 to 100 then  $\sqrt{1200}$  must be changed to  $10\sqrt{12}$  or  $20\sqrt{3}$  or  $5\sqrt{48}$ . Using  $20\sqrt{3}$ , the error in the decimal expression is multiplied by 20; using  $5\sqrt{48}$ , the error is multiplied only by 5. Hence the simplest form is the one that has the smallest coefficient and the largest radicand.

On the other hand, if the class is solving quadratic equations one pupil may use the formula and find that

$$x = \frac{2 + \sqrt{24}}{2}$$

while another completes the square and finds that

$$x = 1 + \sqrt{6}.$$

To show that the answers agree we say that we simplify  $\sqrt{24}$ . If we say *reduce the radicand* we ought to add "without altering the value of the expression." We mean "write in some other equivalent form that will accomplish our purpose whatever that may happen to be in this problem." Algebra abounds in exercises in which we cannot state exactly what we want without using long and complicated directions. On tests I am inclined to use the directions "Do whatever you think should be done" or "Write in some other form—any form that proves you have learned something during the last week."

Let us examine some of the operations which a pupil is taught to do and which at times are not desirable.

To solve  $4(x-3)+5=29$  we may perform the indicated multiplication, but to solve the set

$$4(b-c) = 28$$

$$3(b+c) = 27$$

it is better to divide by 4 and by 3 respectively.

In most problems  $\frac{1}{2}\sqrt{2}$  is more useful than  $\frac{1}{\sqrt{2}}$  but the reverse is true if I want  $(\frac{1}{\sqrt{2}})^{10}$  as in finding the sum of 10 terms of a geometric progression.

One-half of 3 is  $1\frac{1}{2}$  in a butcher shop, but  $\frac{3}{2}$  is better than  $1\frac{1}{2}$  if I wish to square one-half of three. (When I once made this remark in class, a pupil informed me that in a butcher shop "three halves is not the same as  $1\frac{1}{2}$ . When buying bacon, one phrase means three half-pound packages, and the other means a pound and a half cut from a slab of bacon.")

In grammar school a pupil is taught never to leave an answer in the form  $\frac{20}{3}$  but to change it to  $6\frac{2}{3}$ . But, unless it is the answer to a problem so that nothing further needs to be done to it, the form  $\frac{20}{3}$  is usually better than  $6\frac{2}{3}$ . When solving quadratics by completing the square, a pupil often changes  $8+\frac{9}{4}$  to  $\frac{41}{4}$ , then to  $10\frac{1}{4}$ , and then in a later step to  $\frac{41}{4}$ . In grammar school  $\frac{8}{10}$  is frowned on; but  $\frac{8}{10}$  is not the same as  $\frac{4}{5}$  if I am asked to measure a line to the nearest tenth.

When adding two fractions and using  $(a+b)(a-2b)$  as a common denominator, there is nothing to be gained by requiring that the multiplications be performed in the denominator unless the pupils need some additional practice in multiplying. I cannot recall a single instance in which a factored denominator needs to be expanded.

To check the solution of a literal equation it is wise to change  $\frac{a^2-ab}{a-b}$  to  $a$ ; but if this fraction is to be added to  $\frac{a^2+ab}{a+b}$  it would be foolish to make the change.

To divide  $a^2+ab-2b^2$  by  $a-b$  we can use long division or we can factor the dividend; but if the dividend is  $a^2-ab-2b^2$ , factoring would be a waste of time. The simplest or best form of many quantities depends on what is to be done with them. Perhaps after teaching any operation we should give this advice to the pupil: Now that you know how to do this work, don't do it unless you can see some good reason for doing it.

**96. Learning Several Methods.** (Continued from October, 1944.) When a pupil says "Do I work it like page 280 or like page 170?" a good answer is "Do it both ways, and be ready to

explain what difference it makes" or "Do whatever you can do, provided only that you do it correctly" or "When in doubt, do what the signs tell you to do." However, algebra is replete with problems in which it is foolish to do what the signs command and much wiser only to indicate the operation. Algebra seems at times to be an immoral subject which little children should not be allowed to study. Mathematicians often find it profitable not to do what they are told to do, to put off doing things they are asked to do in hopes of avoiding the task; and sometimes the greatest reward goes to the one who does least.

Many of the expressions which resolutions and curriculum-constructing-committees taboo are exactly those which are needed when we allow a pupil to follow the above advice instead of insisting that certain problems must be worked like page 280 and others like page 170. If we permit freedom, then when a pupil presents his solution at the board, many of the class cannot follow his solution unless they are fairly skillful with, what I call, algebraic juggling. As a simple example, consider how many ways a pupil may solve and check

$$\frac{x}{a} - \frac{1}{ab} = \frac{x}{b}.$$

(Must we clear of fractions first? May we transpose first? Need we clear of fractions at all?)

Or, consider such a simple thing as long division. The exercise asks that  $x^2 - 2xy + 4y^2$  be divided by  $x + y$ . And almost 100% of the class does the work correctly. Later ask the class to divide  $-2xy + x^2 + 4y^2$  by  $y + x$ . Some pupils will rearrange the dividend and divisor, dividing  $x^2 - 2xy + 4y^2$  by  $x + y$ . But suppose some pupil prefers to divide  $4y^2 - 2xy + x^2$  by  $y + x$ , and wants to know how he could have obtained a different result. And another pupil may use the quantity as given and have  $-2x$  for the first term of his quotient. And some pupil wants the teacher to give an example in arithmetic corresponding to these peculiar antics of  $x$  and  $y$ . Such embarrassing situations can be avoided by insisting that "this problem must be worked this way, and that problem in that way, and don't ask why."

It may be argued that the thought-provoking situations and the encouragement of a variety of methods is very good for the brighter classes but not for dull classes since they may waste time, distract attention, and send the class off on tangents. From my experience, I offer this suggestion: The homework, if

the pupil does any nowadays, should involve only such simple situations and should be so well started in class that the pupil comes to class with his homework completed and almost wholly correct. If it requires more than five or ten minutes of discussion, then the preparation for it on the previous day was inadequate or the assignment too difficult. Before starting the preparation of the next day's work the class will enjoy the diversion offered by a few peculiar exercises or some more difficult exercises or a discussion of some different methods of working. We can permit the dull pupils to relax for ten minutes while we attempt to stimulate some of the other pupils.

**97. For the Class in Pedagogy.** Prospective teachers are all required to take courses in Methods. Their textbooks include a variety of questions at the end of each chapter, and these can usually be answered by a careful reading of the text. But in the classroom the teacher may be confronted with questions unlike any found in the books. The following situation has arisen often enough to be interesting:

The class is solving quadratics by completing the square. A pupil at the board is solving  $x^2+3x=8$ , and writes next  $x^2+3x+9=8+9$ .

When attention is called to the fact that he forgot to divide 3 by 2, the pupil changes the equation to  $x^2+\frac{3}{2}x+9=8+9$  or, perhaps, to  $x^2+\frac{3}{2}x+\frac{9}{4}=8+\frac{9}{4}$ .

When someone objects and says that the second term should be  $3x$ , the pupil at the board says, "Didn't you tell me to divide 3 by 2?" and writes again  $x^2+3x+9=8+9$ .

If someone says that we should divide 3 by 2, we are back again to  $x^2+\frac{3}{2}x+\frac{9}{4}=8+\frac{9}{4}$ .

The question is: How should the error be corrected?

An unacceptable answer is: This would never happen if the class had had the proper preliminary drill in completing the squares of  $x^2+4x$ ,  $y^2+5y$ ,  $r^2+6r$ , and so forth. We must assume that such drill has been had and that the pupil worked such problems correctly.

I submit two more questions:

When we were studying Installment Buying in a class in Essential Mathematics, a girl asked, "After you have made the down payment, do you have to make the rest of the payments?"

And a boy once asked, "Why do we have different formulas for the areas of rectangles and triangles and circles? Wouldn't it be simpler if we just had one formula for all of them?"

## EASTERN ASSOCIATION OF PHYSICS TEACHERS

### ONE HUNDRED FIFTY-EIGHTH MEETING

Brookline High School, Brookline, Massachusetts

Saturday, December 9, 1944

Joint Session with

The New England Biological Association  
and

The New England Association of Chemistry Teachers

- 
- 9:45 Business Meeting and Election of Officers.  
10:00 Address of Welcome. Ernest A. Caverly, Superintendent of Schools, Brookline.  
10:15 Address: "Recent Developments in Physics," Professor Royal M. Frye, Boston University.  
10:55 Address: "The Physical-Chemical Characteristics of Some of the Proteins of Human Plasma," Dr. L. E. Strong, Harvard Medical School.  
11:40 Address: "Dehydration of Foods in Relation to Feeding the Fighting Forces," Professor S. C. Prescott, Mass. Institute of Technology.  
1:00 Luncheon, Brookline High School.

Meetings of the Biological and Chemistry Associations will be held in the afternoon.

Members are reminded that the Eastern Association of Physics Teachers was founded in 1895, and that 1945 marks the fiftieth year of the Association. Some members have thought it might be a good idea to have some observance of this in the meetings that follow this one. Please bring your suggestions to the business meeting and express your opinions in the matter, for or against.

#### BUSINESS MEETING

The Nominating Committee reported, submitting the following names for ballot.

President: Hollis D. Hatch, English High School, Boston, Mass.

Vice-President: Clarence W. Lombard, High School, Hyde Park, Mass.

Secretary: Carl W. Staples, High School, Chelsea, Mass.

Treasurer: Albert R. Clish, High School, Belmont, Mass.

Executive Committee: Chairman, Everett J. Ford, English High School, Boston, Mass.

The above were elected as officers for the ensuing year.

Mr. Clish extended an invitation to the Association to hold the next meeting at Belmont High School. It was voted to accept the invitation.

#### RECENT DEVELOPMENTS IN PHYSICS

Professor Royal M. Frye of Boston University gave a summary and brief history of some of the recent discoveries and developments in physics.

THE PHYSICAL-CHEMICAL CHARACTERISTICS OF SOME  
OF THE PROTEINS OF HUMAN PLASMA*Abstract of the Address by DR. L. E. STRONG*

The subject of blood plasma is difficult. It involves on the borderline the ideas of the physicist, it applies the ideas of chemistry, and its problems are biological. It is a complicated field, work in which depends on the cooperation of many people, and the field of war medicine. Scientists all over the country, too many to list, are working on it.

The work was started partly by the Rockefeller Institute, and partly by Harvard University. The people directly connected with it, and working practically full time on the problem, number over fifty. The blood has always been recognized as very important. By the electrolytic apparatus of Tiselius, it can be shown graphically how complex the blood is. Dr. Strong first explained this method with the aid of lantern slides. It is based on the fact that proteins move in an electric field. Using a strong light and a suitable optical system, a series of moving boundaries, representing protein moving through the solvent may be observed. The Töpler schlieren (shadow) phenomenon is used to project on photographic film patterns which can be analyzed. From these the concentration of the protein moving with each velocity can be measured.

The plasma shows at neutral reaction that the albumen moves fastest, with alpha, beta, and gamma globulin next in that order. The removal of fibrinogen is marked by the disappearance of a certain peak in the pattern.

Comparison of the shadow patterns of human, bovine, and plasma from other animals was shown. That most resembling the human in pattern was the bovine.

Each of the fractions represented in the patterns is not a single protein but a complex group of many proteins.

The next part of the problem was to find means of separating the fractions without bringing about permanent changes in the chemical nature and the physico-chemical properties of the proteins. Studies and experiments were first carried out using amino-acids which are simpler in structure, but which differ one from the other in sensitivity and properties.

Solubilities in glycerol, water, and ethanol solutions were tried, as well as solutions of calcium chloride, sodium chloride, and sodium sulphate. Each amino acid showed a variety of effects. Proteins showed different properties. Finally, at low temperatures and making use of proper ethanol concentrations, it was found possible to separate the plasma into five fractions, without affecting appreciably the water-solubility and other properties of the proteins. The fractions were fibrinogen (least soluble) albumen, alpha, beta, and gamma globulins.

This method adapted itself to large scale production, and made possible the preparation of whole plasma, or any one of the fractions which might be needed for use in certain particular cases.

A description of the large scale process followed, including preparation of the plasma by centrifuging to remove solid blood components, frac-

tionation of the plasma after removal of the fibrinogen when desired, sterilization, dehydration, packaging, and preservation.

Discussion of various suggested blood substitutes, and comparison of these with plasma followed. The address was illustrated with many lantern slides.

Reference: *Chem. Rev.* 28: 395-415. A large number of references for further reading are appended to this reference.

### DEHYDRATION OF FOODS

*Abstract of Address by* PROFESSOR S. C. PRESCOTT

For some time supplies of rubber, tin, and other materials used in the commercial canning of foods have been curtailed, making it difficult to transport foods. This has made many problems in connection with food supplies.

The last speaker told about blood plasma. Other types of organic substances consist of cells and contain fluids which are somewhat similar. Physics and chemistry are involved, physics entering into the problem of dehydrating foods, chemistry into the study of the materials, and the substances themselves are mostly biological.

The drying of food products is probably the oldest method of food preservation, because it existed naturally before man became civilized, and before he had learned to apply the process himself. Foods dry naturally on plants, as in the cases of peas, beans, nuts, and certain cereals. Drying, then, is a natural process which man has imitated, and in certain ways has carried further than nature.

In a shortage of tin, rubber, and ships, food preservation must be applied more extensively. The processes which are most important have been drying, heat treatment, and steaming. In France 150 years ago Apère at the request of Napoleon's headquarters, studied the problem, and was awarded a prize for his work. Since then refrigeration processes have developed. About the time of the Civil War, food began to be preserved on a large scale by placing it in tin cans, cooking with steam under pressure, and sealing. Before that the process was carried on only on a small scale making use of glass containers.

The drying of foods like cod-fish, apples, pumpkin, etc., for domestic use was the start of dehydrating foods, and, in this country was learned from the Indians. Drying processes for foods are mentioned in the Bible and by Pliny.

Dehydration, however, is a modern art, and started seriously at the time of the first World War. It makes use of the same physical principles as drying, but under controlled conditions, which make possible a better product, and the retention of nutritive qualities, etc.

The old process of drying apples on a large scale at the time of the Civil War was carried out in kilns having perforated floors through which circulated hot air from a furnace. The California fruit-drying industries started in this way.

Now fruits, vegetables, meats, eggs, fruit-juices, milk, and other materials are treated by dehydration methods. Whether this will continue after the war is a question. Dehydrated vegetables, until near the end of the last war were very poor in quality, and even then not comparable with those produced now.

At the beginning of the last war the Surgeon General's Office began the study of foods. One of the principle problems was dehydration, and the importance of humidity, temperature, etc., in the preservation of foods. After the war there was a slump in interest because the products were so poor that nobody would take them by preference. The men who had used them did not want any more. Now they recognize the fact that they are pretty good although not as good as the fresh foods.

Four years ago a revamping process began in the Department of Agriculture, which did not take a great part in the last war. It did not follow up dehydration as it did aspects of canning, etc., which perhaps brought greater revenues to certain politicians.

Only about 40,000 tons of dried vegetables were sent to France during the last war. Foods then were fair, but a horrible example of food technology.

The process of dehydration came in for a complete revision. The Department of Agriculture and a number of educational institutions, including the Massachusetts Institute of Technology, Massachusetts State College, the Oregon Agricultural College, and some others began working on the problem in a small way. They began to study the processes of dehydration, and found out the quantities required, and the conditions the food would have to stand.

In 1944 some of the quantities prepared were:

Apples.....	9,000,000 lb. (before dehydration)
Carrots.....	5,000,000 lb.
Onions.....	10,000,000 lb.
Sweet Potatoes.....	22,000,000 lb.
Soup.....	5,000,000 lb.
Eggs.....	100,000,000 lb.

Dried beef and pork in small pieces for stews, finished stews, corned beef hash, dried orange and lemon juice, and dried milk were some of the other substances prepared. About one fourth of the crop of white potatoes was dried. The total material dehydrated was two and one half billion pounds.

The drying process takes out on the average about 90% of the weight. This would leave about 10% of the weight of raw meat, for example. It also reduces the volume of the material to a little more than 10%, and enables one ship to carry the amount ordinarily carried by seven ships transporting fresh material.

It is the duty of the Quartermaster's Corps to supply all these materials. A large proportion went to the army. The proportion was 15% for civilian use, 30% for lend-lease, and the remainder for the army. That going to the

army was packed in waterproof, gas-proof, insect-proof containers, which could be dropped on the beach or in the sea, or carried on muleback or otherwise. Each package was pretty nearly a balanced diet, a 10-1 ration package. Not all the food in these packages was dried, but some was canned. The vegetable content was reduced to about  $\frac{1}{3}$  the original weight, saving about 70-90% of the vitamins, and made a very satisfactory diet, but one got tired of it.

Professor Prescott showed a map of the United States on which red and green dots marked the distribution of dehydrating plants, the color showing the nature of the product. Some of the geographic divisions were:

New England.....	potatoes and cranberries
New York, New Jersey, Pennsylvania, and Maryland.....	vegetables
The South.....	sweet potatoes
The West.....	beets and cabbages
The Northwest.....	fruits and potatoes
California.....	fruits

There were very few plants in the Central Mississippi region, because in this section other products such as wheat, corn and cotton compete with the products ordinarily dehydrated.

When brought to the dehydrating plants, the raw materials are fairly high in nutrients. Vitamins gradually disappear on storage, so the material must be treated rapidly. The materials are washed, sliced, tested for enzymes which would cause break-down, and dehydrated. The equipment makes use of conveyor systems. After dehydration, the products are tested for moisture content, and for enzymes that would break down the flavor, color, etc.

Various materials including special tough papers, plastics and aluminum foil are used to make the containers water-vapor proof. Inert gases such as nitrogen or carbon dioxide are used sometimes with materials such as carrots, because the latter contain an oil which oxidizes, giving a stale odor or taste.

The containers are very carefully made. They must be insect proof, as insects in the South Pacific area eat through almost anything. A layer of sandpaper in some of the containers has been found to discourage some insects. Most of the materials thus get to the points to which they are sent.

The army food is scientifically planned by dieticians in Washington. Drying reduces the volume, and this is still further reduced by compression. Idaho white potatoes, compressed at 200 lb. per square inch at 110°F. are reduced from shredded dry material 40% in volume.

A package of dried onions compressed at 500 lb. per square inch sounds like a piece of wood when dropped, but soon reabsorbs water. When placed in water it comes back nearly to full normal size and color. No way has yet been found to bring back to 100% lost water.

Professor Prescott showed numerous samples of dehydrated foods.

## PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON

State Teachers College, Kirksville, Mo.

*This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.*

*All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution as on the following pages.*

*The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.*

### SOLUTIONS AND PROBLEMS

**Note.** Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.
2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.
3. In general when several solutions are correct, the ones submitted in the best form will be used.

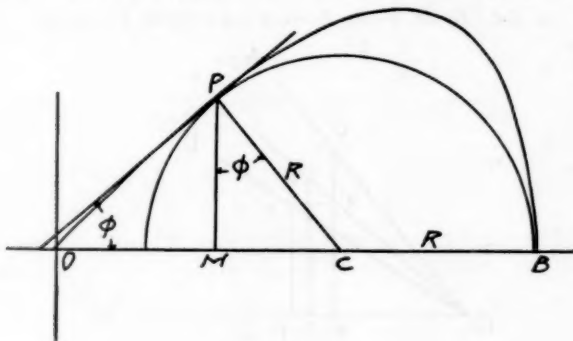
### LATE SOLUTIONS

1897, 1901, 5, 7. M. Kirk, Media, Pa.

1908. Pvt. Milton Schiftenbauer, Camp Wolters, Tex.; George Marsh, East Romulus, N. Y.; Nettie Darling, Milwaukee, Wis.; Ruth P. Waggoner, Schenectady, N. Y.; Joseph L. Wiley, Syracuse, N. Y.; Bro. Felix John, Philadelphia, Pa.; Dorothy C. Hand, Clark Summit, Pa.; O. Marinoff, Denver, Colo.; Edgar A. Rose, Rochester, N. Y.; Wm. A. Richards, Berwyn, Ill.; Grace N. Williams, Cape Girardeau, Mo.; W. I. Chernofsky, Brooklyn, N. Y.; Marvin Kornbluh, Brooklyn, N. Y.

1909. Proposed by Helen M. Scott, Baltimore, Md.

Find the radius of the circle inscribed in the loop given by  $a^2y^2 = x^2(a^2 - x^2)$ .



$$y = \frac{1}{a} (a^2 x^2 - x^4)^{1/2} \quad (1)$$

$$\frac{dy}{dx} = \frac{a^2 - 2x^2}{a(a^2 - x^2)^{1/2}} = \tan \phi. \quad (2)$$

Equation of circle:

$$[x - (a - R)]^2 + y^2 = R^2 \quad (3)$$

in which substitute

$$y^2 = \frac{x^2(a^2 - x^2)}{a^2}$$

$$R = \frac{x^3 + ax^2 - a^2x + a^3}{2a^2} \quad (4)$$

$$\tan \phi = \frac{CM}{y} \quad (5)$$

$$CM = a - x - R$$

$$= \frac{a^3 - a^2x - ax^2 - x^3}{2a^2} \quad (6)$$

$$\frac{CM}{y} = \frac{a^3 - a^2x - ax^2 - x^3}{2ax(a^2 - x^2)^{1/2}} = \frac{a^2 - 2x^2}{a(a^2 - x^2)^{1/2}}. \quad (7)$$

This reduces to

$$x^3 - \frac{a}{3} x^2 - a^2x + \frac{a^3}{3} = 0. \quad (8)$$

(8) has three roots  $a/3$ ;  $-a$  and  $+a$ , but the latter two reduce  $y$  to zero.  
Hence

$$x = \frac{a}{3}.$$

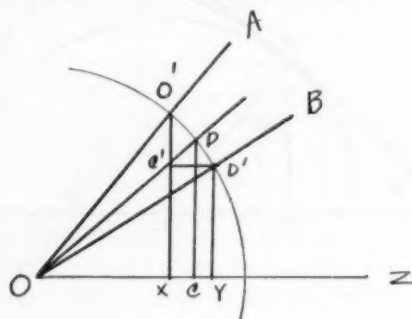
This value in (4) gives:

$$R = \frac{11}{27} a.$$

1910. Proposed by Hugo Brandt, Chicago, Ill.

Prove the identity by use of geometry:

$$\sin \frac{1}{2}(A+B)(\sin A - \sin B) = \cos \frac{1}{2}(A+B)(\cos B - \cos A).$$



*Solution by U. Alfred, Napa, Calif.*

Let angles  $A$  and  $B$  be measured from a common initial line. Extend their terminal sides to cut the unit circle. The bisector of the angle formed by these terminal sides is the terminal side of angle  $(A+B)/2$ .

In the accompanying figure, triangles  $OCD$  and  $O'C'D'$  have their corresponding sides perpendicular. Hence they are similar. Accordingly

$$OC:CD=O'C':C'D'$$

or

$$OC \cdot C'D' = CD \cdot O'C'.$$

Now

$$OC = \cos \frac{1}{2}(A+B)$$

$$C'D' = OY - OX = \cos B - \cos A$$

$$CD = \sin \frac{1}{2}(A+B)$$

$$O'C' = O'X - D'Y = \sin A - \sin B.$$

Therefore it has been proved geometrically that

$$\sin \frac{1}{2}(A+B)(\sin A - \sin B) = \cos \frac{1}{2}(A+B)(\cos B - \cos A).$$

A solution was also offered by the proposer.

**1911.** *Proposed by Chas. P. Louthan, Columbus, Ohio.*

Find the length of the arc of the parabola  $y^2=qx$ , which is intercepted between the points of intersection of the parabola and  $y=kx$ .

*Solution by Brother Felix John, Philadelphia, Pa.*

1. The points of intersection of the given parabola and straight line are

$$(0, 0) \text{ and } \left( \frac{q}{k^2}, \frac{q}{k} \right).$$

2. Differentiating  $y^2=qx$  with respect to  $y$ ,  $x' = \frac{2x}{q}$ .

$$\begin{aligned} 3. \quad s &= \int_0^{q/k} [x'^2 + 1]^{1/2} dy = \int_0^{q/k} \left[ \frac{4y^2}{q^2} + 1 \right]^{1/2} dy \\ &= \frac{2}{q} \int_0^{q/k} \left( y^2 + \frac{q^2}{4} \right)^{1/2} dy \\ &= \frac{1}{q} \left[ \frac{q}{k} \sqrt{\frac{q^2}{k^2} + \frac{q^2}{4}} + \frac{q^2}{4} \log \left( \frac{q}{k} + \sqrt{\frac{q^2}{k^2} + \frac{q^2}{4}} \right) - \frac{q^2}{4} \log \sqrt{\frac{q^2}{4}} \right] \\ &= \frac{q}{2k^2} \sqrt{k^2 + 4} + \frac{q}{4} \log \frac{2 + \sqrt{k^2 + 4}}{k} \\ &= \frac{q}{4} \left( \frac{2\sqrt{k^2 + 4}}{k^2} + \log \frac{2 + \sqrt{k^2 + 4}}{k} \right). \end{aligned}$$

**1912.** *Proposed by Hugo Brandt, Chicago, Ill.*

Find the sum,

$$\sum_1^n \frac{n}{(n+1)!}.$$

*First Solution by Aaron Buchman, Buffalo, N. Y.*

If the series expansion for  $e^x$  which is convergent for all values of  $x$ , is multiplied by  $(x-1)$ , there results

$$e^x(x-1) = (x-1) + x(x-1) + \frac{x^2(x-1)}{2!} + \frac{x^3(x-1)}{3!} + \frac{x^4(x-1)}{4!} + \dots$$

Clear the right side of parentheses, combine similar terms, and

$$e^x(x-1) = -1 + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{3x^4}{4!} + \dots$$

This series converges for all values of  $x$  and in particular for  $x=1$ . Thus

$$0 = -1 + \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots$$

and

$$1 = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots = \sum_1^{\infty} \frac{n}{(n+1)!}$$

*Second Solution by Proposer.*

Let the sum be

$$x = \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \quad \text{Adding}$$

$$e + x = 1 + \left[ 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right]$$

$$e + x = 1 + e$$

$$x = 1.$$

A solution was also offered by Alan Wayne, Flushing, L. I., New York, N. Y.; U. Alfred, Napa, Calif.

1913. *No solution has been offered.*

1914. *Proposed by Paul H. Renton, West View, Pa.*

Prove that the time in which a man could cross a road of breadth  $c$ , in a straight line with the least velocity possible, between a stream of vehicles of breadth  $b$ , following at intervals of  $a$  at velocity  $v$  is

$$\frac{c}{v} \left( \frac{a}{b} + \frac{b}{a} \right).$$

Let  $\theta$  be the angle made by the man's line of travel with the line directly across the road. Then the man must cover a distance  $b \sec \theta$ , while passing through the line of flow of the vehicles. The time he has for doing this is

$$t = \frac{a + b \tan \theta}{v}.$$

Hence the man's velocity would have to be:

$$V = \frac{vb \sec \theta}{a + b \tan \theta}.$$

To make this a minimum find  $dV/d\theta$  and set it equal to zero. This leads to the equation:

$$a \tan \theta + b \tan^2 \theta - b \sec^2 \theta = 0$$

or

$$\tan \theta = b/a.$$

The total distance across the road is  $c \sec \theta$ . The time required to cross this distance would be in general:

$$T = c \sec \theta \frac{(a + b \tan \theta)}{vb \sec \theta} = \frac{c}{vb} (a + b \tan \theta).$$

On substituting the angle for minimum speed, one obtains:

$$T = \frac{c}{v} \left( \frac{a}{b} + \frac{b}{a} \right).$$

*Solution by V. Alford, Napa, Calif.*

### HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

### PROBLEMS FOR SOLUTION

1927. *Proposed by Howard D. Grossman, New York City*

Show that an array of  $N \cdot N$  squares contains  $1^2 + 2^2 + 3^2 + \dots + N^2$  rectangles, of which  $1^2 + 2^2 + 3^2 + \dots + N^2$  are squares.

1928. *Proposed by Hugo Brandt, Chicago, Ill*

Show that

$$\sum_1^n n^3 + \sum_1^n n^7 = 2 \left[ \sum_1^n n \right]^4$$

1929. *Proposed by Norman Anning, University of Michigan.*

Construct a homogenous function of 3rd degree of  $\sin A$ ,  $\sin 3A$ ,  $\sin 4A$ , which shall be identically zero.

1930. *Proposed by Milton Schiftenbauer, New York City.*

Prove that the sum of 13 consecutive integers can not be a perfect square.

1931. *Proposed by C. R. Perisho, McCook, Neb.*

Construct a line perpendicular to the base of a triangle that will bisect the area of the triangle.

1932. *Proposed by Felix John, Philadelphia, Pa.*

Solve the equation

$$\sqrt[m]{(a+x)^2} + 2\sqrt[m]{(a-x)^2} = 3\sqrt[m]{a^2 - x^2}.$$

## BOOK REVIEWS

NAUTICAL MATHEMATICS AND MARINE NAVIGATION, by S. A. Walling, Senior Master R. N. (Ret); J. C. Hill, Education Department, Cambridge University Press; C. J. Rees, Ph.D., Professor of Mathematics, University of Delaware. Cloth. Pages x+221. Answers. 14×21 cm. 1944. The Macmillan Company, New York. Price \$2.00.

This book is designed for use in self teaching by an adult seaman or by a youth preparing for a life at sea. It is divided into two parts. Part I deals with the elementary mathematics basic to marine navigation. This includes not only "the four rules" (the fundamental operations) with integers, fractions, decimals, measures, and percentage, but much problem material in which the "flavor of the sea" predominates. The commonly used metric units and foreign money are reviewed. Stress is placed on ratio and proportion, square roots, and graphs. Formulas are used throughout even though they have no separate section.

Part II, about half the book, is devoted to Marine Navigation. In a setting of practical problems one finds the geometric principles of navigation, the use of Mercator Charts, Magnetic Compass and its corrections, vector triangles in tidal streams to get true heading for a required track, and running "Fix." The sextant is used in "fixing," but not for celestial navigation, which is not included. The book has a thoroughness, a simplicity, and a readability that bespeaks for it a fine reception. Any geometry teacher will be fascinated by its problems and the excellent yet simple geometry which it contains. Any youth of average intelligence, industry, and persistence should be able to master these elementary principles and vocabulary of Marine Navigation.

H. C. CHRISTOFFERSON

ALIGNMENT CHARTS, CONSTRUCTION AND USE, by Maurice Kraitchik, Professor of Mathematics, New School for Social Research, New York. Cloth. Pages 6+94, 16×23.5 cm. 1944. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York. Price \$2.50.

"This book is an exposition of the methods and techniques involved in the construction of nomographic charts, the type known as alignment charts." The author was engaged in the construction of nomographic charts for a period of 25 years on the staff of Lofina, Brussels. The science set forth in this book was developed by Messrs. d'Ocagne and Soreau. These facts are taken from the author's own statements.

The reviewer is very much of a novice with nomographs as they are here so interestingly and stimulatingly presented in concise and rigorous form. The nomographic chart is a diagram or combination of diagrams for the graphic presentation of a mathematical law. It is a time saver when the same equation has to be solved repeatedly. The slide rule is really one such nomographic chart in which one scale slides over another. Nomographic charts bring the speed, efficiency, and accuracy of a slide rule into many complex relationships. For a more extended treatment of the theory the author refers to d'Ocagne's "Traité de Nomographie." His own treatment is "elementary and concise."

The simplest form of nomogram is a conversion scale such as that for converting temperatures from Fahrenheit to Centigrades, or the square root, cube root, trigonometric, and logarithmic scales of a slide rule. These involve only two variables:  $Y = X^2$ ,  $a = \log b$ ,  $F = f(c)$ . An alignment chart for a function of three variables consists of three scales so constructed that

a straight line connecting three points one on each scale determines a triple of values which satisfy the given function.

$$\text{E.g. } ab=c, ab=c^2, \frac{1}{c} = \frac{1}{a} + \frac{1}{b}, c^3+ac+b=0, v=(1+t)^n.$$

One of the most important types of alignment charts has three parallel lines as base scales. The author proposes such charts for solving by alignment such complex relations as—

$$W = \frac{M}{560\pi d^2}, \quad I = \frac{\pi}{64} (D^4 - d^4).$$

By means of parallel scales, not equally distant,

$$X = A \frac{0.209}{0.209 - i}.$$

can be solved by alignment. Functions involving four or more variables yield themselves also to this almost instantaneous solution. Enough has been said to intrigue the reader, if he is curious, into many happy hours with this stimulating volume of 90 pages.

H. C. CHRISTOFFERSON

**SENIOR MATHEMATICS**, by Harl R. Douglas, Director of the College of Education, University of Colorado, formerly a teacher of Mathematics at the University of Missouri, H. S. and Lucien B. Kinney, Acting Dean of the School of Education at Stanford University, formerly a teacher of Mathematics in St. Paul, Minnesota. Cloth. Pages x+438. 14×20.5 cm. 1945. Henry Holt and Company, Inc., 257 Fourth Avenue, New York, N. Y. Price \$1.52.

"Senior Mathematics is an integrated selection of procedures for solving the more common mathematical problems of shop, business, military life, travel, science, home, health, and safety. Its primary aim is to develop mastery of fundamental mathematics." So say the authors in their preface. It is designed (1) as a terminal course for students who have had neither geometry nor algebra and (2) as a refresher for others, even for adults out of school.

The authors begin with a minimum of traditional drill in computation, functional drill being brought in with problems which need computation. Much checking on mastery of fundamental skills and much practice material is spread throughout the 400 page book. The first 30 pages deal largely with linear measurement but contain tests in whole numbers, fractions, and decimals and the student who does poorly on these is referred to the remedial section in the last 64 pages of the book for needed additional practice.

The stress on problem solving is well done and most of the problems used are real and practical. Such statements as  $\frac{1}{4}$  inch and 6 mm. for saw cuts should not be too annoying. Emphasis on problems with "missing data" is excellent for stressing careful reading and analysis. The slide rule is introduced early (p. 86) and used in several places throughout the book thus distributing practice with it. The authors select much useful mathematics from arithmetic, algebra, geometry, and trigonometry and apply it to a variety of real situations. The drawings are very well done, e.g. the vernier, and the descriptions given for similar technical situations are amazingly clear considering their condensation. "Starred" exercises are provided for the bright, ingenious, and industrious student.

"Senior Mathematics" reveals a wealth of excellent materials, much careful research, and an abundance of wide reading. Yet a critical reviewer might feel that the book is too ambitious in its dual purpose objective, that it is too voluminous for seniors with no mathematics, that it appears too much like a consensed dictionary useful largely for those who have once learned the materials and need refreshing. It seems difficult to justify, even as starred exercises for the gifted few, the attempt to teach by rule the operations with negative numbers in 4 pages of text, quadratic equations solved by formula in one and a half pages of text, the binomial theorem for computing compound interest, arithmetic and geometric progressions and even logarithms. These traditional units seem entirely unnecessary, and can contribute little except confusion and frustration for a large portion of those for whom the book is designed. The authors display real courage and insight in their informal selection of material, in their skill in presenting the formula, the slide rule, and scale drawing, and in their starred exercises for the bright pupil. Yet such starred exercises as the above (logarithms are not starred) can contribute little "enrichment," in the opinion of the reviewer, to the major purposes of the book.

As a whole this book is an outstanding contribution to mathematical education. It reveals the way mathematics functions in a wide variety of situations and does it in such a way that the ability to use a mathematical technique such as a formula can transfer to a number of situations. A skillful teacher will need to omit wisely questionable "enrichment," to select carefully, and to supplement generously in order to "develop mastery of fundamental mathematics." The book will be most useful as a stimulus to teachers who need help in making mathematics live and function.

H. C. CHRISTOFFERSON

THE PHILOSOPHY OF BERTRAND RUSSELL, edited by Paul Arthur Schilpp, *Northwestern University*. Cloth. Pages xvi + 815. 15 × 23 cm. 1944. 101 Fayerweather Hall, East, Northwestern University, Evanston, Ill. Price \$4.00.

This is Volume V of the *Library of Living Philosophers*, of which Doctor Schilpp is Editor. Like its predecessors, it aims to present the interpretations and criticisms of a wide range of a particular thinker's scholarly contemporaries, each of whom is "given a free hand to discuss the specific phase of the thinker's work which has been assigned to him." All of the contributed essays are submitted to the philosopher whose work they discuss, in order that he may prepare replies to them for inclusion in the volume.

The present volume opens with Russell's own description of his mental development, in which he traces briefly his "intellectual journeys," indicating the sources of his interest in history, mathematics and philosophy. This short autobiographical sketch is followed by 21 essays on as many aspects of Russell's philosophy, arranged in order from the abstract to the concrete; they begin with logic, the scientific method, theory of knowledge, and psychology and metaphysics; these are followed by discussions of ethics and religion, political and social philosophy, and finally the philosophy of history.

Each of the contributors assumes a considerable knowledge of Russell's own writings on his assigned topic, and the reader without such first-hand acquaintance will frequently find himself handicapped. The references to Russell's voluminous writings are very complete, however, and anyone who wishes to go to the source can easily do so. None of the essays is in any way popular, and some of them are by no means easy reading; but they are all presented by scholars whose claim to be heard is unquestioned.

Not the least valuable portion of the book is Russell's reply to his critics. With some of what some of them say he professes to be in agreement, but frequently he finds that they have misunderstood him. This he deplors because he has always tried above anything else to be clear and to say exactly what he means. His reply therefore gives him an opportunity to try to set right at least some of these misunderstandings and misconceptions. For the student of philosophy it is comforting to realize that even a great philosopher cannot always understand a colleague and that even a master of prose cannot always make himself understood. This chance to see the interplay of various minds is one of the valuable fruits of this volume and of the others of its series.

The book is not to be picked up to while away an hour; nor is it likely that a real understanding of even one aspect of Russell's philosophy can be gained from reading it. But as a guide to study it should be very valuable.

The volume is excellently printed and bound. It contains what is intended to be a complete bibliography of all of Russell's writings, and a full index.

JOSEPH D. ELDER  
Wabash College

FUNDAMENTALS OF ALGEBRA, by Joseph Nyberg, *Hyde Park High School, Chicago, Illinois*. Cloth. Pages vi + 336. 1944. American Book Company, Chicago.

This is a brief text, which presents the content usually included in a first course in algebra. The first half of the book takes up the study of graphs, formulas, equations and problems. The second half treats the more formal operations and is intended to develop skill in their use. This arrangement conforms to the recommendation frequently made that the practical parts of the course should come first and the more formal topics later. However, the author points out that the content is adapted to a rearrangement of chapters if the teacher wishes.

Other features of the text include fifteen pages of tests, discussions on the importance of mathematics in vocations, algebraic exercises preceded by similar arithmetical exercises, and provision for individual differences by the inclusion of special sets of exercises.

This text contrasts rather sharply with the more voluminous books which have been appearing during the past ten years. It is an attempt to omit non-essentials and to push students along with the essentials of the subject. The number of exercises is entirely adequate for any class. Verbal problems are emphasized and extensive practice is provided. The text is a desirable addition to the field.

G. E. HAWKINS

TODAY'S GEOMETRY, by David Reichgott and Lee R. Spiller, *The New Haven High School, New Haven, Connecticut*. Cloth. Pages xvi + 400. Revised, 1944. Prentice-Hall, Inc. New York.

In the teaching of geometry there are three fairly distinct schools of thought on the subject. We may think of one approach as the traditional one. Most textbooks on the subject are quite similar in content and method and emphasize deductive proof, geometric information, and applications. The vast majority of teachers conform to that practice. A second approach that has been advocated by various individuals in recent years suggests that the study of logical reasoning or the nature of proof should be the primary objective of the course. A third approach recommends that

acquiring geometric information and ability to apply it in problems be the major objectives of the course.

The authors of *Today's Geometry* subscribe to the third approach and have some basis for their feeling that their 1938 edition anticipated many of the emergency requirements advocated by educational authorities. The text definitely does not stress formal proof. Many of the usual theorems are stated as postulates and some are developed inductively. The proofs to theorems are given, and some of the exercises call for proof. However, the main objectives are to impart geometric information and to provide applications from a variety of sources.

The authors have included material intended to modernize and to make their text more useful. In the chapter on locating points (loci) they include the topics: distances on a sphere; time; time zones on the earth; the 24-hour system; and global maps. They have a chapter on vectors and air navigation. One chapter of about 35 pages is devoted to the measurement of solids and one chapter reviews the essentials of arithmetic and of elementary algebra.

The illustrations and drawings are well chosen and presented attractively.

The book contains a wealth of applications of geometry. It obviously is a valuable addition to the list of textbooks in geometry.

G. E. HAWKINS

FOURIER SERIES, by G. H. Hardy and W. W. Rogosinski. Paper. 100 pages. 14×21 cm. 1944. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$1.75.

As stated in the preface of this work, it is the purpose of the authors to present a treatment of Fourier series that is "written in a modern spirit, concise enough to be included in this series, yet full enough to serve as an introduction to Zygmund's standard treatise." The entire treatment is theoretical in character, and presupposes on the part of the reader a certain foundation of knowledge in functions of a real variable, including the elements of Lebesgue integration. It is indicated in the preface that the actual knowledge presupposed is contained, for example, in Chapters X-XII of Titchmarsh's *Theory of Functions*. An excellent set of notes at the end of the work contains brief comments on various omitted topics, together with references for the reader. The tract is written in an excellent style, and is complete in itself; it is not written for beginners, however, and it is highly advisable that the reader have some previous knowledge of the subject.

The following brief summary indicates the titles of the individual chapters, and an abbreviated list of topics therein treated. I. *Generalities*: preliminary definitions; functional spaces  $L^p$ ; orthogonal systems in  $L^2$ . II. *Fourier Series in Hilbert Space*: the Riesz-Fischer and Parseval theorems, with related results. III. *Further Properties of Trigonometrical Fourier Series*: the Riemann-Lebesgue theorem; results on the order of magnitude of Fourier coefficients; integration of Fourier series; series with coefficients which are positive and decreasing; preliminary discussion of the Gibbs phenomenon. IV. *Convergence of Fourier Series*: various criteria for the convergence of a Fourier series, together with the associated criteria for the convergence of the conjugate series. V. *Summability of Fourier Series*: regular methods of linear summation of series, with particular emphasis on so-called  $K$ -methods; discussion of summability of Fourier series, and of the conjugate series, by first order Cesàro means and by the Abel (Poisson) method. VI. *Applications of the Theorems of Chapter V*: example of a Fourier series which diverges almost everywhere; convergence

properties of particular sub-sequences of partial sums; strong summability of Fourier series; further discussion of the Gibbs phenomenon; theorems on the existence of the conjugate function, and the connection between the convergence of a Fourier series and the convergence of its conjugate series. VII. *General Trigonometrical Series*: a discussion of the Riemann theory of trigonometrical series, terminating with the theorem that if a trigonometrical series converges, except on a countable set, to a finite and integrable function  $f$ , then it is the Fourier series of  $f$ .

W. T. REID

Northwestern University

GENERAL METEOROLOGY, by Horace Robert Byers, Sc.D., *Professor of Meteorology, The University of Chicago*. Cloth. Pages x+645. 14×22.5 cm. 1944. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, N. Y. Price \$5.00.

During the rush of war training there have appeared on the market many books dealing with meteorology. Some of these have been for the express purpose of training pilots in the rudiments of weather forecasting so that they might interpret a forecast and be able to apply it to their own flight problems, or so that they might even be able to anticipate the course of the weather from their own observations. This type of book is often more of a syllabus of methods and rules without much explanation of the underlying theory. Other books have been written for the training of meteorologists and have gone so deeply into the theory of weather analysis that they were comprehensible only to those versed in advanced thermodynamics.

This new book by Dr. Byers seems to anticipate the peace time need of a more general text that is between the two technical extremes mentioned above. With the probable popularity of private aviation after the war there will be an ever increasing demand for a general understanding of the weather. It would seem then that there should be a place in the college curriculum for the study of general meteorology along with the present general courses in astronomy, geology, biology, chemistry, and physics. The text at hand is for just such a course. It gives the fundamental principles of weather analysis along with the modern developments in synoptic meteorology. The theoretical discussions of radiation, heat balance and thermodynamics of the atmosphere are of such a nature that they would be best understood by one with a background of general physics and the integral calculus. However, the development starts from fundamentals so that one may learn the necessary physics from the discussions given.

The practical applications of importance to the aviator are discussed in the latter chapters on fog, ice formation, and thunderstorms. In general, the descriptive material of the text is quite understandable without the mathematical derivations of some of the more technical portions. Accompanied with laboratory work, with instruments and weather charts, the text might well be used for the special training of pilots. It may also be used as an introductory text for those anticipating more advanced study in meteorology.

The text is well illustrated with photographs of cloud types and instruments as well as with graphs and charts that illustrate the principles presented. Ample appendices supply the International Code of symbols used in transmitting weather information and for plotting it on charts. Many graphical aids are also given so that solution of many equations may be easily read.

H. R. VOORHEES,  
Chicago City Colleges, Wilson Branch

**MATHEMATICS, INDUSTRIAL SERIES**, by John W. Breneman, C. E., Associate Professor of Engineering Mechanics, Pennsylvania State College. Cloth. Pages xii+224. 13×21 cm. 1944. McGraw-Hill Book Company, Inc., 330 W. 42nd St., New York, N. Y. Price \$1.75.

This book is one of a series known as the Industrial Series prepared under the direction of the Division of Engineering Extension, Pennsylvania State College. It is a revision of the author's first edition of the book published in 1941. It is written for the purpose of providing an adequate study in mathematics needed in industrial work for the person whose formal schooling in this field has not been completed and to provide suitable material for those who wish to refresh their knowledge in the subject by home study.

The book includes such chapter titles as Common Fractions, Decimal Fractions, Powers and Roots, Areas and Volumes of Simple Figures, Circular Speeds, Tables, Formulas, Fundamentals of Algebra, Equations, Simultaneous Equations, Geometrical Constructions, Trigonometry, Oblique Triangles, Logarithms, and Solutions of Triangles by Logarithms. Percentage is included in the chapter on decimal fractions. The three chapters devoted to algebra is a good coverage of the subject as it is used by the industrial worker. Trigonometry includes in addition to right triangles a good development of solutions of oblique triangles and solution of triangles by logarithms. The careful presentation of the subject of powers and roots includes the square root of decimal and common fractions. There are fifty pages of mathematics tables given in the appendix. These are reprints from *Carnegie Pocket Companion*, by Carnegie Illinois Steel Corporation. A very good explanation of the use of these tables is given in the text. The 471 problems provide an abundance of practice material and the unique plan of numbering the problems consecutively through the book make lesson assignment quite easy.

DOYLE T. FRENCH,  
Elkhart High School

**MAN IN THE AIR**, by Herbert S. Zim. Cloth. Pages x+332. 13.5×20.5 cm. 1943. Harcourt, Brace and Company, 383 Madison Avenue, New York, N. Y. Price \$3.00.

In the foreword the author has this to say: "The Story of *Man in the Air* touches a crucial point in flying: the point where it affects man's body and mind. It is also crucial because no plane is better than its pilot and because planes can already stand conditions that the human body cannot endure. Man's use of aviation cannot develop faster than the limitations of his body permit. Plane designers must consider the weak point in flying—the pilot. Already much progress has been made in adapting the plane to man and training man to use the upper air. The lessons of the war will have a profound effect on post war flying."

A book on applied physics, chemistry and biology of the body of the pilot, it treats of the limitations of the body of the pilot. There are twenty-two chapters of very interesting reading which is important to those who want to know the limits of endurance of a pilot. Two serious errors occur in the chapter on "Change in Speed and Direction." The formula

$$F = \frac{v^2}{20s} \quad \text{should be} \quad F = \frac{v^2}{2gs},$$

and the two problems that follow are incorrectly solved because of the incorrect formula. The book is rich in facts and applied scientific laws and

principles which make valuable material for reports in high school science classes. It is a book that bears re-reading.

ESTIL B. VAN DORN,  
Washington High School  
Indianapolis, Ind.

A START IN METEOROLOGY, by Armand N. Spitz, *In Charge of Department of Meteorology, The Franklin Institute, Philadelphia, Pennsylvania and Instructor in Meteorology, Air-Mar Navigation Schools*. Cloth. 95 pages. 13×19.5 cm. 1942. The Norman W. Henley Publishing Company, 17 West 45th Street, New York, N. Y. Price \$1.50.

A book on meteorology for young people who wish to prepare themselves for useful careers in aviation. It is a book which has been planned, prepared, produced and published at high speed. It does not pretend to be an exhaustive treatise but one which hits the high points of interest in meteorology. It is full of rough hand sketches of fundamental concepts explained in the text. Definitions of meteorological terms are simply and interestingly told in story form and then questions are asked about these terms and answered. The questions and answers at the end of the book cover the important facts that a beginner should know about meteorology, providing he cares to go farther into the subject. On page 86 is a bibliography of important texts in meteorology classified as second course and advanced. This is one book in meteorology that a high school student can read and understand because it is written for young people.

ESTIL B. VAN DORN

VITALIZED FUNDAMENTALS OF ELECTRICITY (IN GRAPHICOLOR), by Robert H. Carleton, *Head of Department of Science, Summit High School, Summit, New Jersey*. Paper. Pages viii+184. 12.5×19 cm. 1944. College Entrance Book Company, 104 Fifth Avenue, New York, N. Y. Price 60 cents less 25% to schools.

The purpose of this book is to follow pre-induction training course as outlined in the War Department Manual PIT-101 and as recommended to the high schools by the U. S. Office of Education. It is low in cost, small in size, and is intended to be a student's permanent guide and stand-by whenever a memory refresher may be needed.

The outstanding features of this little book are the two-color, red and black, diagrams which are very attractive as well as descriptive of definite underlying physical principles. The text is also full of formulas in the second color. Typical problems are solved and the important steps of the solutions are in color for sake of emphasis. The selection of content is to provide basic knowledge for boys who will soon operate machines and electrical devices in modern warfare. The concluding chapters on alternating current theory, sound and wave phenomena, and radio fundamentals are simply written and well-illustrated. Another outstanding value of this book is the material that teachers can use in making objective tests and problem assignments. The answers to the problems are given and are correct to three significant figures. The student is also given Best Answer Tests, Questions, and Completion Tests at the end of each of the eleven units of study. The Appendix contains a discussion of vectors, an outline of topics, and a well organized index.

ESTIL B. VAN DORN

VITALIZED FUNDAMENTALS OF MACHINES (IN GRAPHICOLOR), by Robert H. Carleton, *Head of Department of Science, Summit High School, Summit,*

*New Jersey.* Paper. Pages vi+186. 12.5×19 cm. 1944. College Entrance Book Company, 104 Fifth Avenue, New York, N. Y. Price 60 cents less 25% to schools.

The purpose, outstanding features and types of tests and problems are very similar to those of the book on *Vitalized Fundamentals of Electricity*. I know of no other text which so clearly teaches that the mathematical formulas are expressions of scientific thoughts. Each typical problem is a method of thinking through a fundamental law or principle in the most simple way it can be done. Emphasis is placed on solving most problems by using a proportion. The diagrams of machines are very attractive in two colors, red being used to highlight and distinguish basic ideas from other necessary but less important details. About one half of the book is given to the study of machines and the other half to the subject of heat. Heat is studied as a form of energy which is used to run machines. The unit on the automobile and the one on the airplane show them to be complex machines made up of the simple machines studied in the preceding units. The book is a very useful help to physics students in reviewing for tests and examinations that they may have to take when they are inducted into the armed forces of U. S.

ESTIL B. VAN DORN

ODD NUMBERS, OR ARITHMETIC REVISITED, by Herbert McKay. Cloth. 215 pages. 12.5×19 cm. 1943. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price \$2.00.

This is not a textbook but when you sit down to read it you should have a pencil and paper or be prepared for mental gymnastics in the field of arithmetic. The topics are fifteen in number as follows: Millions, Billions and Trillions; Great Powers and Little Powers; How We Got Logarithms; Proportion; Comparisons; Proportion in Triangles; Weights and Measures; The Delusive Average; Approximations; Multiplication and Division; Tables; Units; Oddities of Number; The Construction and Solution of Problems; and Scales Notation.

The author is undoubtedly English, as the billion which he discusses in the first section is a million million and his finance problems are in English monetary units. It will be impossible to discuss all the units or even most of them in this brief review. In trying to show the size of a quantity such as a million he introduces small cubes, one-tenth inch on a side arranging them first in linear units, then in square units, and finally in cubes. Throughout the book the solar system is used to make comparisons in distances, areas, and volumes.

The story of "How We Got Logarithms" is very interesting. Also of particular note is the section on "Weights and Measures," a comparison of the English and metric systems. For the common measurements the former is, according to the author, better suited in most respects than the latter and consists of more convenient units. Whether you agree with him or not his logic is very interesting.

The problems which he includes in "The Construction and Solution of Problems" are unusual, and for the most part are trick problems whose solution is not difficult when all factors are considered.

The book uses only arithmetic and simple algebra. It is not just a book for mathematicians, but is for the ordinary individual who likes to deal with numbers, first because it is practical to do so, and second because they are interesting for their own sake. I can truly say that I enjoyed reading it and can recommend it to anyone who desires several hours of reading which will require some mental exercise upon the part of the reader.

ALBERT R. MAHIN  
Hartford City, Indiana